1. The vertical equation of momentum is given as:

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} - g = \frac{1}{\rho} \frac{\partial p}{\partial z}
\]

Where the first four terms on the LHS of the equation are the advection terms, \( g \) is the gravitational acceleration, and the term on the RHS of the equation is the pressure gradient force.

a.) Starting with the vertical equation of momentum one can arrive at the hydrostatic equation (Hint: You can assume the atmosphere is at rest and horizontal accelerations are weak). Show, stating all assumptions, how this done. Explain the significance of this equation.

b.) Now using the hydrostatic equation, the ideal gas law, and 1st law of thermodynamics given below:

Derive the dry adiabatic lapse rate (show all work):

\[
\Gamma d = \frac{\partial T}{\partial Z} = -\frac{g}{C_p} \sim 10\text{K/Km}
\]

c.) Why doesn’t the air temperature actually drop according to this lapse rate, if for example you started to climb Timms Hill (highest point in WI).

2. Please explain the difference between a reversible and irreversible process. Give/explain an example of each.

3. The second law of thermodynamics implies the existence of another variable of state called entropy, usually denoted by "S". Define entropy and write out what it means mathematically. For an adiabatic process, \( \dot{s} S= \). Is this process reversible or irreversible? For a reversible process at constant volume \( \dot{s} S= \)? For a reversible process at constant pressure \( \dot{s} S= \)?

4. Now the specific enthalpy of a gas is defined by: \( H = U + P \alpha \), where \( H \) is enthalpy, \( U \) is internal energy, and \( P \alpha \) is work.

a.) Prove that \( \dot{s} H \) for an ideal gas is an exact differential.

b.) Starting from the equation for specific enthalpy of a gas above, obtain the equation for \( C_P \) (heat capacity of air) under constant pressure: ( \( \dot{s} h/\dot{s} T \))\( p = C_p \)

c.) Calculate the change in enthalpy of a unit mass of dry air as it is compressed adiabatically from an initial pressure of 70kPa and temperature of 10C to final pressure of 100kPa.