Downslope Wind Storms
How does acceleration over the wing affect pressure field?

Equation of Motion for frictionless flow:
\[ \frac{\partial \mathbf{V}}{\partial t} = \mathbf{f} - \nabla k - \alpha \nabla p + \mathbf{g} \]

If we assume a horizontally homogeneous, hydrostatic reference state where subscript "o":
\[ \alpha_o \frac{\partial p_o}{\partial z} = -g \]

and so, subtracting:
\[ \frac{\partial \mathbf{V}}{\partial t} = \mathbf{f} - \nabla k - \alpha_o \nabla p + \alpha' / \alpha_o \mathbf{g} \]

where primes are deviations from base state. Integrate equations of motion for frictionless flow along the trajectory of the flow (\( \cdot ds \)):
\[ \int_s \frac{\partial \mathbf{V}}{\partial t} \cdot ds = \int_s \mathbf{f} \cdot ds - \int_s \nabla k \cdot ds - \int_s \alpha_o \nabla p' \cdot ds - \int_s \left( \frac{\alpha'}{\alpha_o} \right) \mathbf{g} \cdot ds \]

Allow only for flow in x/z plane, assume Lamb vector is small (1: 0).

and flow is steady state \( \frac{\partial \mathbf{V}}{\partial t} = 0 \), and then we get Bernoulli's equation

\[ \frac{1}{2} \left| \mathbf{V} \right|^2 + \left( \frac{\alpha'}{\alpha_o} \right) gz + \int_s \alpha_o ds p \]
Flow Mountain Ridge

- Infinitely long mountain, no flow around ridge
- Consider first an airplane wing:

\[
|\alpha' / \alpha_o g| = |\alpha_o \partial p' / \partial s|
\]
Small Ridge

• Similar to airplane wing:

\[ \left| \alpha' / \alpha_0 g \right| = \left| \alpha_0 \partial p' / \partial s \right| \]
Meso-Beta Scale Ridge

- Resonant gravity response is involved, low pressure shifts increasingly down stream as the scale of the ridge becomes larger:

\[ \frac{\alpha'}{\alpha_o g} : \left| \frac{\alpha_o \partial p'}{\partial s} \right| \]

10 km
Witch of Agnesi Ridge

- Lets consider a “Witch of Agnesi”, bell shaped mountain (normally used for analytical mountain wave studies) having the formula:

\[
z_s = h \frac{a^2}{a^2 + d^2}
\]

- \(a\) is the half-width, \(h\) is the maximum height, and \(d\) is the distance from the ridge top, and \(z_s\) is the topography height.
More about Witch of Agnesi

• Easy for analytical solutions
• NOT a sine wave, is a bell shaped ridge that contains a spectrum of wave components representing many wavelengths
• Some parts of the ridge may be at super Rossby radius scale and some may be at sub-Rossby radius scale for instance
Froude Number

• Important influences on atmosphere response to flow over an object:
  – (a) Length scale of the object
  – (N) Brunt-Vasallai frequency, the vertical stability providing a restoring force for gravity waves:
    \[ N = \left( \frac{g \partial \theta}{\theta \partial z} \right)^{\frac{1}{2}} \]
    – (U) velocity of flow normal to the ridge
Froude Number

• Define Froude Number:

\[ F_r = \frac{\text{inertial frequency}}{\text{Brunt-Vasallai Frequency}} = \frac{U/a}{N} \]
Inertial Cutoff, ie Rossby Number

- The coriolis parameter is another important parameter. If the mountain is big enough, we get lee cyclogenesis, not gravity waves! So we must consider the Rossby Number, ie:

\[ R_o = \frac{a}{L_R} \]
Flow Over a Ridge

- We consider flow over shallow (h << depth of troposphere) ridges of several half-widths and look at the results of a linear analytical solution for the Witch of Agnesi mountain.

- The solution to the linear problem yields a wave equation of the form:

\[ w_{tt} + \left( l^2 - k^2 \right) w_{zz} = 0 \]

- w - vertical velocity
- z - height above surface
- k - vertical wave number
- l - Scorer Parameter
Vertical Wave Number

• $L_x$ is the horizontal wavelength of the gravity wave. This parameter in the vertical wave equation is purely nonhydrostatic!

$$k = \frac{2\pi}{L_x}$$
Scorer Parameter

- This parameter is related to the transmissivity of the atmosphere to gravity waves considering only hydrostatic processes

\[ l^2 = \frac{N^2}{U^2} - \frac{1}{U} \frac{d^2U}{dz^2} \]
When Gravity Waves?

• Gravity wave solutions only exist when
  \[ l^2 - k^2 > 0 \]

• Therefore, there is a “short wave cutoff” scale, below which gravity waves cannot exist:
  - \( L_z \) is the vertical wavelength of the gravity wave
  - \( L_x \) is the horizontal wavelength of the gravity wave

\[
L_z = \frac{\sqrt{2\pi}}{\sqrt{l^2 - k^2}} \quad \text{and} \quad L_x > \frac{\sqrt{2\pi}}{l}
\]
Narrow Ridge: Evanescent waves

Fig. 12.3. Perturbation by a narrow typical mountain range (a = L_s/2π = 1 km) in an unlimited uniform stratified current (u = 10 m s⁻¹). Upper part: vertical projection of the streamlines; displacement indicated by small arrows. Lower part: perturbation of pressure and longitudinal wind velocity at ground level. The horizontal displacement is negligible. [From Queney (1948).]
Medium Ridge: Mountain (gravity) waves

Fig. 12.1. Perturbation by a medium-size typical mountain range \((a = L_s/2\pi = 10 \text{ km})\) in an unlimited uniform stratified current \((u = 10 \text{ m s}^{-1})\). Upper part: vertical projection of the streamlines; displacement indicated by small arrows. Lower part: perturbation of pressure and longitudinal wind velocity at ground level. The horizontal displacement is negligible. [From Queney (1948).]
Typical Mountain Wave (Lenticular) Cloud
Double Wave (Lenticular) Cloud
Flying Saucer Wave Cloud
Lenticular Cloud
Broad Ridge:
Lee Cyclogenesis for larger modes, GW for smaller modes

Fig. 12.2. Perturbation by a broad typical mountain range \( (a = L_\phi/2\pi = 100 \text{ km}) \) in an unlimited uniform stratified current \( (u = 10 \text{ m s}^{-1}) \). Upper part: vertical projection of the streamlines; displacement indicated by small arrows. Lower part: horizontal projection of a streamline and of the asymptotic isobar at ground level. [From Queney (1948).]
Medium-Narrow ridge, but with Scorer Parameter (l) varying with height. This “traps” shorter waves of the “Witch of Agnesi” mountain, but transmits vertically the longer ones, leading to lee waves.

- This is mostly a nonhydrostatic effect – why?

- The shorter waves have solutions in low levels where l is large, but do not above, so they reflect off
Lee Waves
Lee Waves
Mountain (Gravity) Waves

- $F_r << 1$; i.e. static stability dominates over inertia
- $R_o > 1$ or $L_x < L_R$; i.e. effect of stability dominates over Coriolis
- $L_x > \sqrt{2\pi/l}$, i.e. scale is larger than short-wave cutoff for gravity waves
Vertically Propagating Gravity Waves

- Wave group velocity transporting energy up and into wind i.e. $u'w'<0$
- Wave phase propagation, down and into wind i.e. $u'w'<0$
- Westerly momentum transported down, i.e. $u'w'<0$

Schematic representation of wave package
Gravity wave absorbed at critical level where phase speed equals wind speed and air statically stable above.
Effect of moisture on Mountain Waves

- Effect is to lessen the Brunt Vasallai frequency because latent heat reduces lapse rate:

\[ N_{\text{moist}}^2 = g \left\{ \frac{1 + \frac{L_{vl} r_s}{RT}}{1 + \frac{\varepsilon L_{vl}^2 r_s}{c_p RT^2}} \left( \frac{\partial \ln \theta}{\partial z} + \frac{L_{vl}}{c_p T} \frac{\partial r_s}{\partial z} \right) - \frac{\partial r_l}{\partial z} \right\} \]
• Increases depth of mountain wave
• Increases horizontal wavelength
• May cause some trapping of shorter wavelengths
Theory of Downslope Wind Storms

- They go by a number of names:
  - Chinook winds (Rockies, Indian name that means “snow eater”)
  - Foehn wind, name used in Europe
  - Santa Ana wind, name used in Southern California
• Downslope wind storms are related to mountain waves
• Mountain waves will locally increase the winds on the lee side of the mountain, but typically not to severe levels
• But in downslope wind cases they get very strong reaching severe levels routinely (> 55 kts)
• Let's look at a famous documented windstorm hitting Boulder Colorado on 11 January, 1972
Fig. 12.8. Cross section of the potential temperature field in degrees Kelvin over the mountains and foothills as obtained from analysis of NCAR Queen Air and Sabreliner data on 11 January 1972. Data above 500 mbar are exclusively Sabreliner from 1700–2000 MST. Data below 500 mbar are primarily Queen Air from 1330–1500 MST. Flight tracks are indicated by the dashed lines, except by crosses in turbulent portions. It is not possible to determine if apparent westward displacement with height of the major features is real or related to the time difference between the two flights. Windstorm conditions on the ground extend eastward to where the isentropes rise sharply, a few miles east of the origin at the Jefferson County airport. [From Lilly and Zipser (1972).]
Fig. 12.9. Contours of horizontal velocity (m s\(^{-1}\)) along an east-west line through Boulder, Colorado, as derived from the NCAR Sabreliner data on 11 January 1972. The analysis below 500 mbar was partially obtained from vertical integration of the continuity equation, assuming two-dimensional steady-state flow. [From Klemp and Lilly (1975).]
Fig. 12.6. West to east vertical cross section of potential temperature across the Sierra Nevada. Dashed line represents sailplane soundings. Observed Chinook arch or Foehn wall cloud is illustrated over barrier crest, as well as rotor cloud at low levels to the east and lenticular cloud at higher levels. [From Holmboe and Klieforth (1957).]
Klemp and Lilly Theory

- Based on hydrostatic simulations
- Partial reflection of group velocity off of tropopause creating resonance
- Need tropopause height to be integer number of half wavelengths above surface
- Resonance increases amplitude of mountain wave...no wave breaking in their hydrostatic theory
Fig. 1.2.10. Numerical simulation of 11 January 1972 case: (a) Displacement of potential temperature surfaces; (b) contour of west wind component (m s$^{-1}$). Maximum surface velocity lee of the mountain is 55 m s$^{-1}$. [From Klemp and Lilly (1978).]
Clark and Pelteir (1977)

- Same effect but upper wave breaks
- The breaking upper wave destabilizes upper troposphere and lower stratosphere ducting the underlying mountain wave more
- Strong amplification of lower troposphere wave
- Critical level at $\frac{3}{4} L_z$ optimal
Fig. 12.11. Potential temperature for the second Boulder, Colorado, windstorm simulation in which the mixing coefficients are determined through first-order closure. Times are (a) 3200, (b) 4160, (c) 5120, (d) 6020, (e) 7040, and (f) 8000 s. Note the extreme deflection of the trononause in the last frame. [From Peltier and Clark (1979).]
Fig. 12.12. Total horizontal velocity field for the Boulder, Colorado, windstorm simulation. Times are (a) 3200, (b) 4160, (c) 5120, (d) 6020, (e) 7040, and (f) 8000 s. Contour interval is 8 m s$^{-1}$. In (f) the horizontal wind maximum in the lee of the peak is in excess of 60 m s$^{-1}$. [From Peltier and Clark (1979).]
Influence of Mid-Level Inversion

• Created by a cold pool to the west and to the east, such as a Great Basin High to west of Rockies and Arctic High to east
• Inversion near or just above ridge top
• Inversion traps wave energy below, leading to large amplification down low and formation of a *hydraulic jump*
Hydraulic Jump Analogy

- Current thinking among mountain meteorologists
- Imagine flow along a rocky stream bed:
  - Water under air is analogous to the layer of cold stable air at the surface under less stable air above! Notice the water waves are trapped from moving upward into the air as the waves in the stable layer of air are trapped from moving upward into the less stable air.
  - When water is much deeper than rocks, turbulence, water flows across the rocks with little turbulence. You could take a boring raft trip down such a laminar stream.
• Now imagine that the water lowers to be just deeper than the rocks. Now you have whitewater! The water plunges down the lee side of the rocks and even digs a little hole, depressing the surface and blowing out rocks etc.

• The same is true for the downslope wind. Trapped beneath the inversion, the wave amplifies and breaks, clowing out Boulder!
Class Case Study: February 3, 1999 west of Boulder

1400 UTC

Wind Speed

Potential Temperature