AOS 610 Fall 2023 Prof. Hitchman

Problem Set #1 10 points Due Tuesday, September 26

Please show your work and staple pages together.

1. (1 pt) *Material derivative*. The change in quantity T following the motion (*Lagrangian derivative*) can be related to the local time rate of change (*Eulerian derivative*) and temperature advection by the zonal wind as $\frac{dT}{dt} = \frac{\partial T}{\partial t} + u\,\frac{\partial T}{\partial x}$. Estimate the rate of temperature change following the motion under the following conditions: a local time rate of change of temperature of -0.36 K/hr, an eastward temperature increase of 1 K/ 100 km, and westerly flow at 10 m/s.

2. (3 pts) Consider *plane Couette flow* for a viscous fluid confined between two flat plates separated by distance b in the y-direction, where the steady state velocity in the x-direction is u = U y/b, U is the velocity of the upper plate relative to the stationary lower plate, and v = w = 0.

   a) Find the rate of *linear strain*, rate of *shear strain*, and *vorticity*.

   b) Defining the *streamfunction* by u = - $\frac{\partial \psi}{\partial y}$ and v = $\frac{\partial \psi}{\partial x}$, express ψ in terms of U, b, and boundary condition ψ_o.

   c) Sketch the *streamfunction*, labeling your axes.

3. (3 pts) *Flow visualization, turbulent diffusion, and advection.*
For this problem you can watch the three videos or you can conduct the experiments personally. Each individual particle of colored aerosol will follow a *trajectory*. The *streak line* or plume of all aerosol trajectories can be useful for visualizing flow around obstacles.

* Watch the first video (or experiment on a day with light wind). The ruler provides a spatial scale and the time is indicated in the film. Remember to use SI units (kg, m, s).

a) What do you think is the cause of the turbulence observed in this plume?

b) Estimate the altitude, δz, of the top of the *convective* plume δt = 2 seconds after lighting, hence the vertical velocity at the top of the plume, $w \sim \delta z/\delta t$.

c) Estimate the width, δx, of the *convective* plume δt = 2 seconds after lighting.

d) Use scale analysis to estimate the turbulent diffusion coefficient K ($m^2\ s^{-1}$) in a simple diffusion equation for smoke particle concentration, n (particles m^{-3}):

$\frac{\partial n}{\partial t} = K\frac{\partial^2 n}{\partial x^2}$. (Hint: In scale analysis you can cancel n on both sides.)

To summarize, the combination of vertical advection and horizontal diffusion can be expressed as $\frac{\partial n}{\partial t} = -w\,\frac{\partial n}{\partial z} + K\,\frac{\partial^2 n}{\partial x^2}$.

* Watch the second video (or experiment on a day with moderate wind).
In a moderate wind the plume may be primarily *advected* sideways, and will expand with time in the vertical direction due to turbulence. This combination can be expressed as $\frac{\partial n}{\partial t} = -u\,\frac{\partial n}{\partial x} + K\,\frac{\partial^2 n}{\partial z^2}$.
e) Estimate the horizontal wind speed u near the front of the plume 2 seconds after lighting.

* Watch the third video (or experiment near an obstacle).
f) Describe the flow phenomenon that you see. You may wish to browse figures in Tritton to help name the phenomenon.

4. (1 pt) *Divergent and rotational flow components.* Show that
a) the divergence of the rotational part of the flow is zero (*nondivergent flow*) and
b) the curl of the divergent part of the flow is zero (*irrotational flow*).

5. (2 pts) *Dynamical similarity.* Problem 18, p. 473 in Tritton.

18. A string of density σ, diameter d, length l is held in tension across a channel through which fluid of kinematic viscosity ν flows. The string vibrates when its fundamental natural frequency coincides with the Kármán vortex street frequency. The following observations were made of the tension F at which resonance occurred for various flow speeds U of a fluid with $\nu = 1.0 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$:

$U(\mathrm{m\,s^{-1}})$	0.2	0.4	0.6	0.8	1.0	1.2	1.4
F(N)	0.096	0.43	1.03	1.92	3.00	4.3	5.8

Suppose now that the fluid is changed to one with $\nu = 5.0 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$. At which of the following speeds can the resonance tension be predicted and then what is it?

$U(\mathrm{m\,s^{-1}})$	0.4	1.0	5.5
		2.4N	90N

(You may assume that the only properties of the string that enter the problem are σ, d, and l, and that changes in the density of the fluid have no effect (except through the kinematic viscosity); and that no harmonics are generated.)