AOS 610 GFD I Fall 2023 Prof. Hitchman Name ___________________________

Problem Set #2 10 points Due Tuesday October 24, 2023

1. (1 pts) *Orbits* An orbit may be defined as occurring when $\vec{g}_{eff} = \vec{\nabla}\left(\Phi_g + \Phi_c\right) = 0$, where the gravitational and centripetal potential functions are Φ_g = G M/r and $\Phi_c = \Omega^2 r^2/2$. Solve for the distances r from the center of the earth for an object going around a) once every 23 hr 56 min (sidereal day) and b) once every 27.3 days.

2. (1.5 pts) *Circulation and vorticity*
a) What is the circulation about a circle of radius 500 km for a cyclonic vortex in the Northern Hemisphere, where the tangential wind speed increases outward at 10 m/s per 100 km?

b) What is the relative vorticity within the circle?

c) Evaluate the vorticity of the earth itself near the north pole, using the same method for a circle of radius 500 km centered at the north pole, assuming that the earth is in solid body rotation and the angular frequency is $\Omega = 7.292 \times 10^{-5}\ s^{-1}$.

3. (1.5 pt) *Thickness*
a) Estimate the 1000-500 hPa thickness for an atmosphere with constant lapse rate 6.5 K/km and surface temperature T_o = 273 K.

b) Examine a 1000-500 hPa thickness chart from a weather analysis or numerical forecast, such as a "surface" chart from the NCEP NAM model, at http://www.aos.wisc.edu/weather/Models. The dashed yellow contours are labeled in decameters (multiply by 10 to get thickness in meters). Estimate the mean 1000-500 hPa thickness across the contiguous 48 states. Using this, estimate the magnitude of a typical thickness anomaly (departure from the mean).

4. (1.5 pts) *Geostrophic wind*
a) Estimate the geostrophic wind at 4 km over Tateno Japan in Fig. 2 of the article about Ooishi and b) compare with the radiosonde observations in Fig. 9 (1° latitude = 111 km).

5. (1 pt) *Thermal wind*
Surface winds are calm in Madison, but temperatures in the lower troposphere are observed to increase eastward at 1 K per 100 km. Estimate the wind speed and direction over Madison at 5 km altitude if the surface temperature is 290 K and the lapse rate is adiabatic.

6. (1 pt) *Curvature versus Coriolis*
Consider the momentum equations on a sphere, with no initial motion. A 10-m diameter steel tube through the center of the earth was recently discovered on Madison's near east side.
a) If you dropped a 1-cm diameter steel ball down the middle of the tube, at approximately what depth would it hit the side wall, and which side of the wall will it hit? Assume that friction is negligible.
b) What would happen if the ball were dropped through a similar tube, but at the equator?

7. (1 pt) *Tracer continuity equation*

Ozone mixing ratio, r, is observed to be 50 ppbv in Madison and the wind is from the northeast at 20 knots. Stations along the shore of Lake Michigan, 125 km to the east, report ozone observations of 100 ppbv. A photochemical sink of 2 x 10^{-4} ppbv s^{-1} applies throughout Wisconsin. Assuming that ozone varies linearly, and only in longitude ($\frac{\partial r}{\partial y} = 0$), forecast the ozone mixing ratio 6 hours from now in Madison. (1 m/s ≈1.94 knots)

8. (1.5 pts) *Ideal gas law*

a) Show that the linearized ideal gas law may be written $\frac{p'}{\bar{p}} \approx \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}$.

b) The Boussinesq approximation implies that $|\frac{p'}{\bar{p}}| \ll |\frac{T'}{\bar{T}}|$, so that $\frac{\rho'}{\bar{\rho}} \approx -\frac{T'}{\bar{T}}$. Evaluate the validity of this assumption for synoptic scale variations by inspecting surface analyses at http://www.aos.wisc.edu/weather/Models and choosing typical mean and anomaly values for pressure and temperature. Make sure you convert to Kelvins for use in the ideal gas law.