AOS 610 GFD I Fall 2023 Prof. Hitchman Name ___________________________

Problem Set #3 Due Thursday, November 16, 2023

1. (1 pt) *Rotating tank*. Considering the radial acceleration of fluid elements accomplished by the height gradient, derive a formula for the dependence of depth, h, on radius, r, for an incompressible fluid in solid-body rotation at angular frequency Ω in a cylindrical tank with a flat bottom and free surface. Let H be the depth at the center of the tank.

2. (1.5 pts) *Static stability*. Starting from the definition of potential temperature, it can be shown that

$$N^2 \equiv \frac{g}{\theta}\frac{\partial \theta}{\partial z} = \frac{g}{T}\left(\frac{\partial T}{\partial z} + \Gamma_d\right)$$

where N is the buoyancy frequency and $\Gamma_d = g/c_p$ is the dry adiabatic lapse rate. Using the attached soundings at Singapore and Green Bay, WI, estimate the rate of change of potential temperature with height $\frac{\partial \theta}{\partial z}$ in K/km and buoyancy period $\tau_B = 2\pi/N$ in minutes, where N is the buoyancy frequency (s^{-1}), for the following regions:

a) tropical lower stratosphere (~ 100 - 30 hPa)
b) tropical upper troposphere (~ 400 - 100 hPa)
c) midlatitude troposphere (~ 1000 - 200 hPa).

3. (1 pt) *Conservation of absolute vorticity*.
A tropical air mass over the Atlantic near 30°N moves northward conserving absolute vorticity.
a) If its initial relative vorticity is zero, what will it be at 60°N?
b) If the air mass has a 500-km radius and is moving in solid body rotation, what are the tangential wind speed and the sense of rotation at 60°N?

4. (2 pts) *Ertel's potential vorticity*.
From Fig. 9.1a in Andrews, Holton and Leovy, estimate zonal mean Ertel's PV at
a) 45°N, 500 hPa
b) 45°N, 100 hPa.
Express your answer in PV units, where 1 PVU = 1×10^{-6} K-m^2-kg^{-1}-s^{-1}. Check your answers against Fig. 9.1b. Note that values in that panel are in units of 1×10^{-5} K-m^2-kg^{-1}-s^{-1}, or 10 PVU.
c) Describe the relationship between PV (Fig. 9.1b) and ozone mixing ratio (Fig. 9.1c) below ~20 km altitude.
d) How do the distributions of ozone and PV differ above 20 km? What causes this difference?

5. (2.5 pts) *Ocean surface layer energetics*. Consider a well-mixed oceanic boundary layer of depth 20 m near the equator. The wind stress is given by $\tau = \rho\, c_d u^2$, where c_d = 0.001, ρ is air density, and u is the wind speed.

a) Assuming that there is no friction between the well-mixed layer and the deep ocean, and that all of the energy goes into kinetic energy of the slab upper ocean, how long would it take for the ocean current v to reach the speed of the air, assuming that u = 5 m/s in the above formula? [Assume a slab of depth h. The momentum change in the slab with net viscous force $\frac{\partial \tau}{\partial z}$ gives $\rho_w$dv/dt = ($\rho\, c_d u^2$- 0)/h.]

b) Consider what happens in this formulation when the current speed reaches v = 5 m/s. How would you modify the relation $\tau = \rho\, c_d u^2$, to take into account the current speed v?

c) When the current speed reaches 1 m/s, what is the rate of generation of kinetic energy per unit mass in the slab? [Multiply your solution in a) by v to form an equation for time rate of change of KE in the upper slab of the ocean.]

d) In steady state one may assume that the energy input by the wind stress is balanced by the viscous dissipation rate per unit mass, η. Using your answer from c), estimate the length scale at which deformation work balances molecular diffusion.

[The Kolmogorov microscale is $l^* = \left(\frac{\nu^3}{\eta}\right)^{1/4}$; use ν for water.]

e) Taking a typical value of c_p for the ocean from Table A3.1 of Gill, what is the rate of change of temperature associated with this kinetic energy input by wind stress? [Use conservation of energy in the presence of viscous dissipation $c_p \frac{\mathrm{dT}}{\mathrm{dt}} = \eta$.]

6. (1 pt) *Eddy diffusivity and eddy mixing length theory*.

a) Using eddy mixing length theory, express the acceleration due to vertical momentum flux convergence, $-\frac{\partial}{\partial z}\overline{u'w'}$, in terms of an eddy diffusivity, K_{zz}, and curvature of the time mean flow, $\frac{\partial^2}{\partial z^2}\,\bar{u}$. [You may assume that turbulence causes departures, u', from the average wind, $\bar{u}$, so that $u = u' + \bar{u}$, and that the time mean vertical motion, $\bar{w} = 0$, so that $w = w' = \delta z/\delta t$. In mixing length theory, vertical displacements, δz, cause eddy zonal winds $u' \approx -\delta\, z\, \frac{\partial \bar{u}}{\partial z}$.]

b) Estimate K_{zz} for turbulence characterized by length scale $L \sim$ 2 m and time scale $\tau \sim$ 1 s.

7. (1 pt) *Differentiating phase*

a) *Propagation*.

Defining streamfunction to be $\psi(x,t) = \psi_o e^{i(kx-\omega t)}$, express $\frac{\partial \psi}{\partial x}$ and $\frac{\partial \psi}{\partial t}$ in terms of ψ.

b) *Growth or decay*.

If $\omega = \omega_r + i\,\omega_i$, where $i = \sqrt{-1}$, what is the dependence of $\psi(x,t)$ on ω_i?

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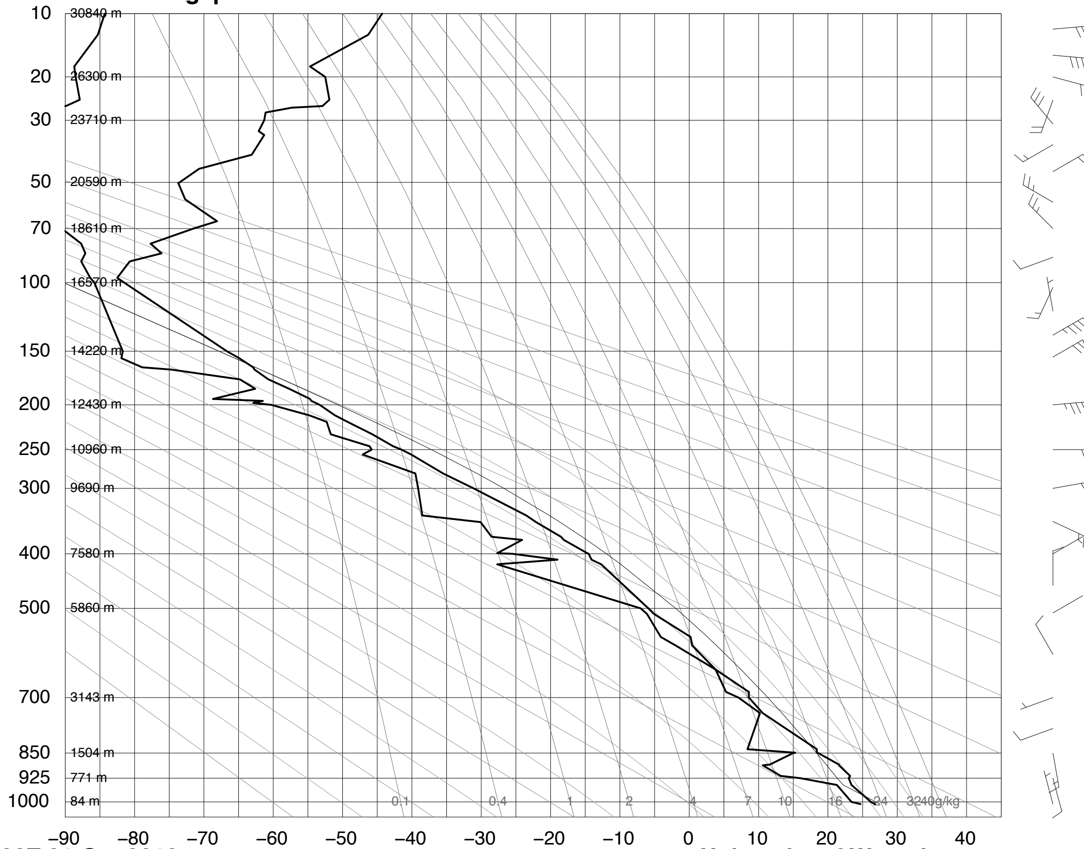

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SLON	103.98
SELV	16.00
SHOW	−0.66
LIFT	−3.95
LFTV	−4.31
SWET	220.0
KINX	38.00
CTOT	20.90
VTOT	24.50
TOTL	45.40
CAPE	1494.
CAPV	1641.
CINS	−50.4
CINV	−19.1
EQLV	161.3
EQTV	161.3
LFCT	823.6
LFCV	861.5
BRCH	2014.
BRCV	2212.
LCLT	295.2
LCLP	944.6
MLTH	300.1
MLMR	18.09
THCK	5776.
PWAT	54.21

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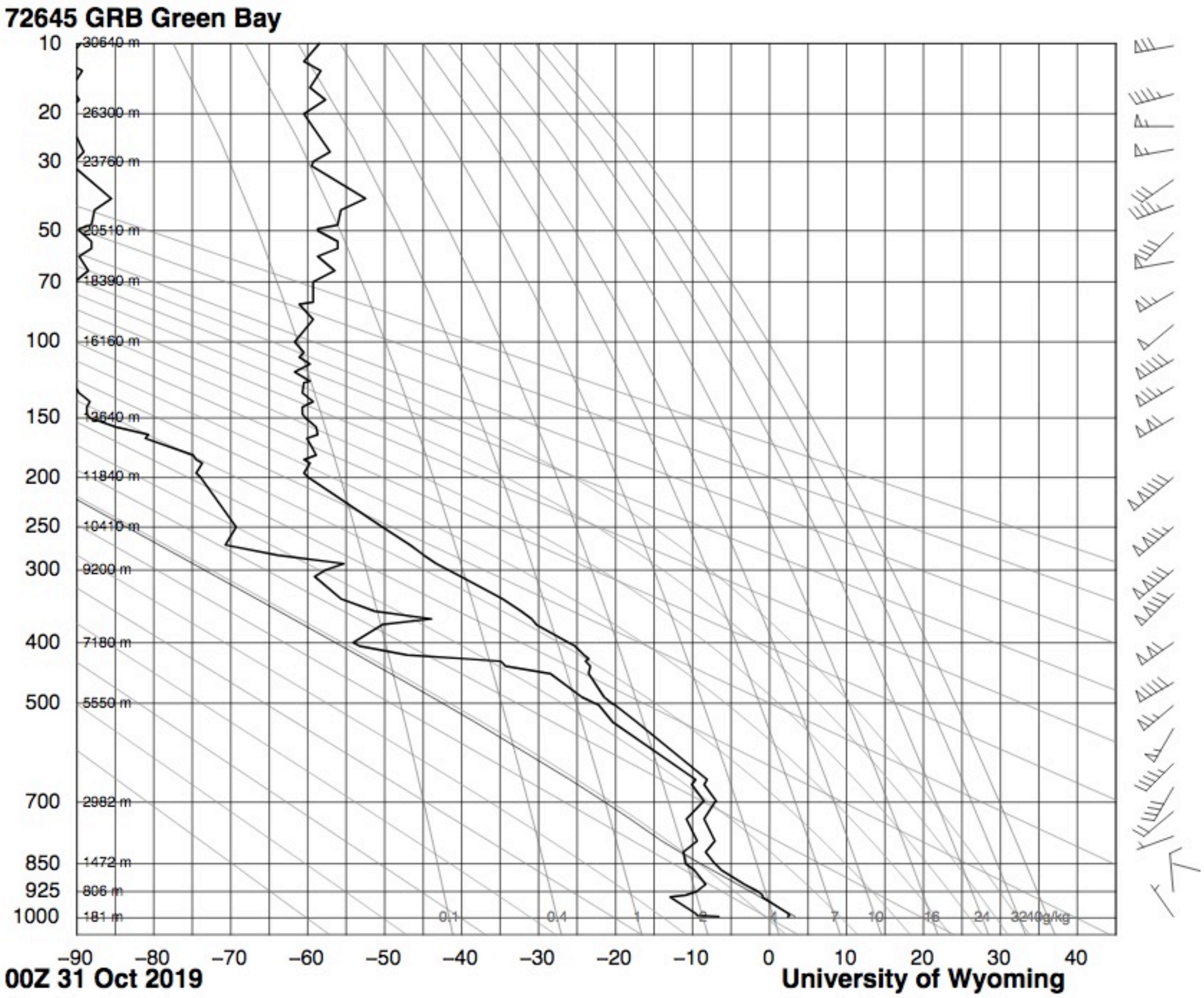
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20
30
50
70
100
150
200
250
300
400
500
700
850
925
1000
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26300 m
23760 m
20510 m
18390 m
16160 m
13640 m
11840 m
10410 m
9200 m
7180 m
5550 m
2982 m
1472 m
806 m
181 m
0.1
0.4
1
2
4
7
10
16
24
32
40g/kg
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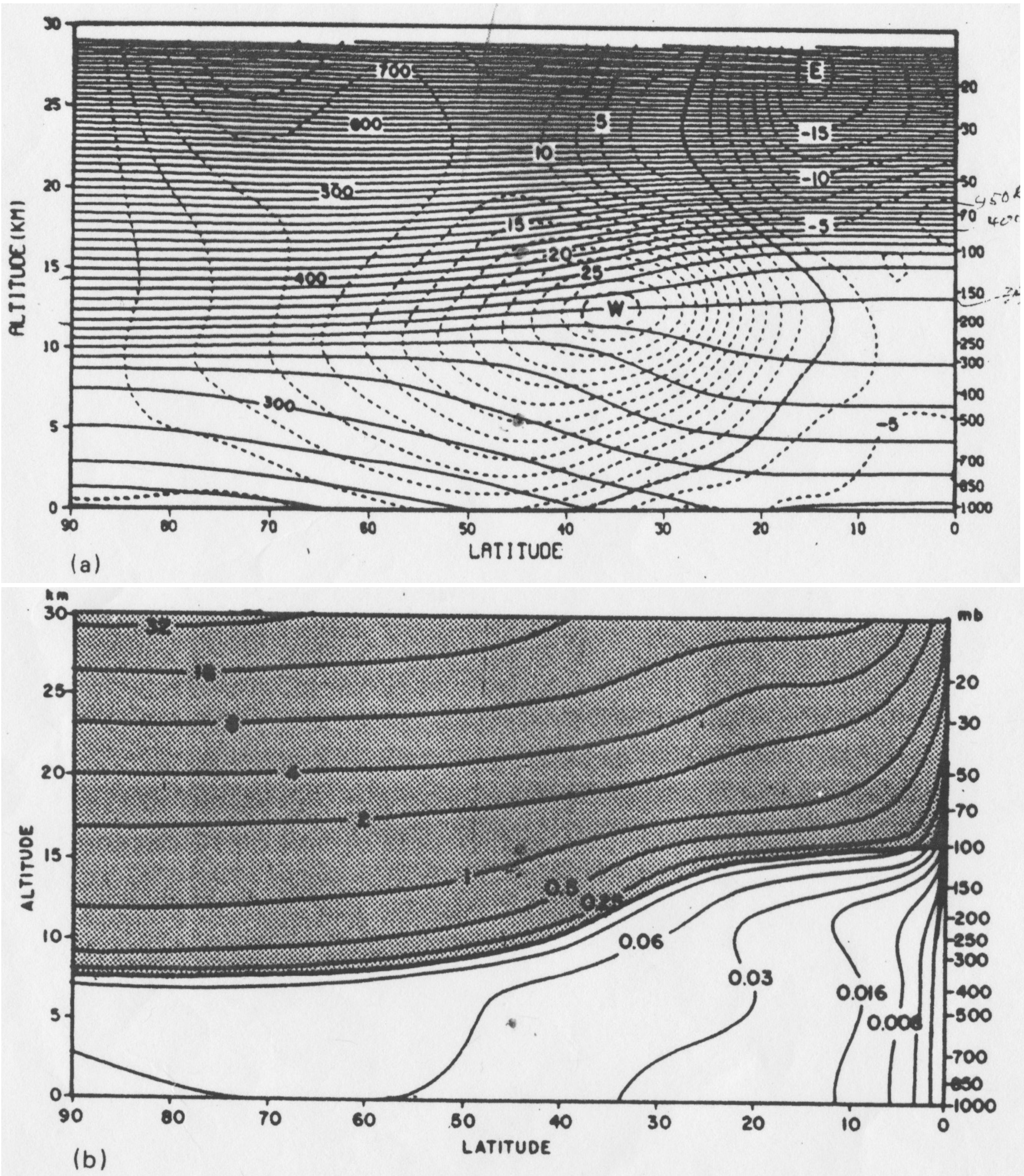
ALTITUDE (KM)
LATITUDE
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600
500
400
300
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10
15
20
25
W
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-15
-10
-5
(a)
km
mb
ALTITUDE
LATITUDE
32
16
8
4
2
1
0.5
0.25
0.06
0.03
0.016
0.008
(b)

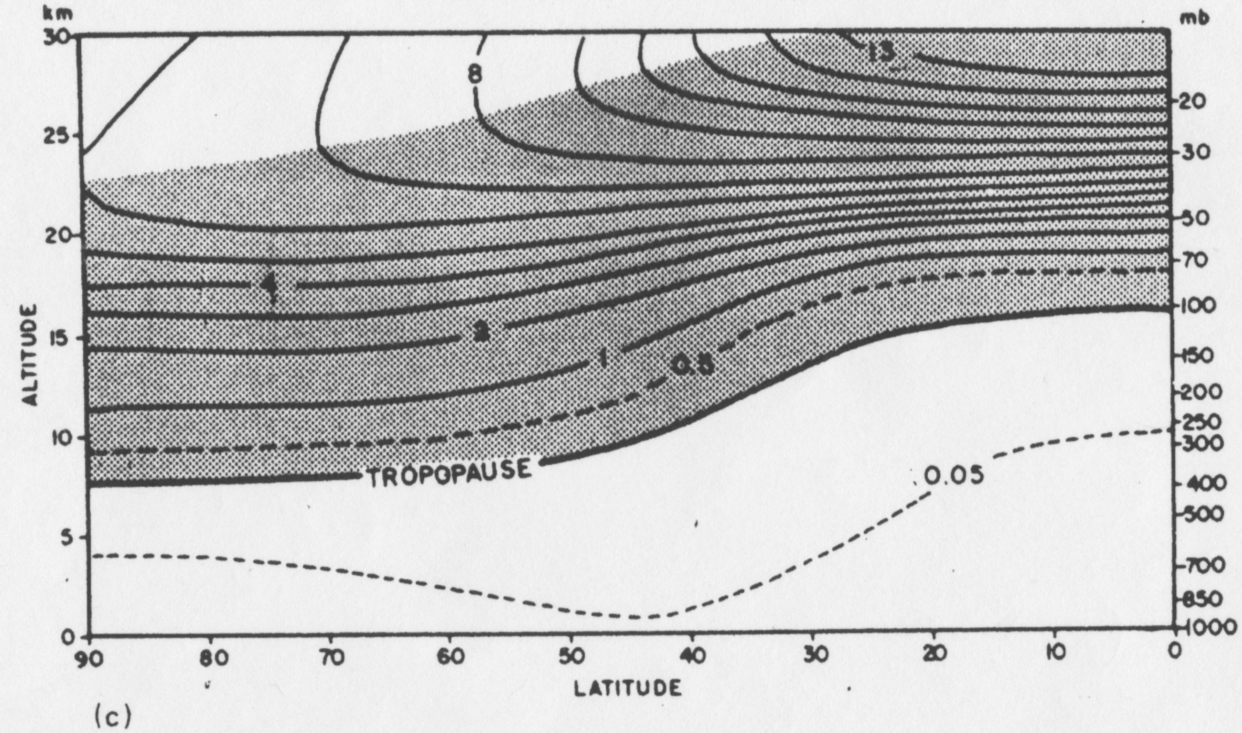

Fig. 9.1. Northern-Hemisphere zonal-annual mean cross sections for some quasi-conservative tracers. (a) Potential temperature (solid contours, kelvins) and zonal wind component (dashed contours, $m\,s^{-1}$). (b) Ertel potential vorticity in units of $10^{-5}\,K\,m^2\,kg^{-1}\,s^{-1}$. Area above the mean tropopause is shaded. (c) Ozone mixing ratio in parts per million by mass (ppmm). Shading extends from the mean tropopause to the level of maximum mixing ratio. Note that the tropopause [marked by heavy lines in panels (b) and (c)] intersects several of the potential temperature surfaces shown in panel (a). [From Danielsen (1985), with permission.]