AOS 610 GFD I Prof. Hitchman

Key concepts on topics from lectures for the first quiz

1. Flow through or past an obstacle

pressure gradient force (PGF; Nt) = prime mover

kinematic viscosity (m^2/s) opposes the prime mover

dynamic viscosity (kg/m-s) = kinematic viscosity x density

Reynolds number

a) ratio of inertia to viscous forces

b) dynamic similarity of flows with same Re

c) captures transition to turbulence

laminar versus turbulent flow

nondimensionalization

2. Poiseuille flow

no-slip condition

steady state

viscous stress (Nt/m2)

viscous force (Nt)

flow curvature required for non-zero viscous force

momentum diffusion to the wall balances PGF

Poiseuille Law enables measurement of viscosity

mass flux = density x speed

entry length turbulence due to abrupt shear

turbulent slugs grow into surroundings

turbulence transports momentum, heat, constituents

more efficiently than molecules

if flow is turbulent, to achieve the same increase

in flow speed a greater increase in PGF is needed

compared to the laminar flow regime (cf. Fig. 2.11

in Tritton)

3. Flow past a cylinder

Re transitions are different than for pipe flow, but still represent

trend toward greater flow complexity

Flow creates a drag on an object, which is reflected in diminished flow

speed in the wake

At very low Re creeping flow involves symmetric streamlines, with

the balance between viscosity and PGF. Viscosity is essential for

drag. Influence of object extends to many L.

At higher Re attached eddies become unstable and are shed downstream

At still higher Re turbulent reattachment of the boundary layer reduces

the size of the wake, thereby decreasing the drag!

Objects introduce vorticity into the flow

At small Re drag is proportional to U

At most values of Re drag is proportional to U\*U (most geophysical situations)

The drag coefficient is an important nondimensionalization of drag

which is used in all ocean and atmosphere models to approximate the

complexity of subgrid scale phenomena

4. Rayleigh Number and convection

Differential heating gives rise to motions

Ratio of buoyancy to viscous forces

Buoyancy force and distance from boundaries promote flow complexity, while

thermal (kappa) and mechanical (nu) dissipation tend to reduce flow complexity

Temperature and density perturbations are of opposite sign

Reduced gravity

Coefficient of expansion

At the critical value Ra=1708, buoyancy overcomes viscous dissipation

and the fluid begins to move. “Benard convection:

At first roll-like structures transport heat, then at higher Ra

structures become more polygonal in shape, with increasing time variations

5. Other diffusion concepts

Prandtl number is the ratio of mechanical to thermal diffusivity (nu / kappa)

There is a turbulent Pr analogue

In water the diffusivity of salt is much slower than for heat, giving

rise to a mixing phenomenon known as salt fingers

Some buoyancy phenomena are dominated by variations in surface tension

with temperature or with chemical concentrations (like soap in “color burst”)

6. Link to chaos theory

For increasing Re or Ra, instabilities lead to turbulence. At low

Re or Ra dissipation creates a stable attractor. At higher

values oscillations can occur with a stable limit cycle. At

still higher values instabilities lead to a Strange Attractor,

where dissipative processses create preferred regions in

phase space, but the trajectory in phase space is unpredictable.

Lorenz (1963) developed his ideas of chaos theory using

the mathematical equations for Benard convection in a box.

7. Flow kinematics

Stress/strain

rate of linear strain

rate of shear strain

vorticity

streamfunction - flow - vorticity (invertibility principle)

rotational or nondivergent flow

velocity potential

divergent or irrotational flow

Laplacian

streamline, trajectory, and streakline

stretching and folding = irreversible mixing

jet or a wake = vorticity dipole

vorticity max = streamfunction min

cyclonic and anticyclonic for NHem and SHem

vortex tube = material surface for adiabatic frictionless flow

conservation of vorticity: without viscosity vorticity cannot be introduced;

vorticity can change following the motion due to vortex stretching or shrinking

Kelvin’s circulation theorem: In the inviscid limit, circulation is permanent, vorticity

is “frozen into the flow”

Types of rotation

rigid body rotation = no viscous force, constant vorticity at all points = 2 omega

irrotational vortex = zero vorticity at each point except the center (minus infinity)

=> shear produces filamentation

acts like angular momentum conserving flow with a mass sink in center

Rankine vortex is a combination of inner solid body rotation

and irrotational vortex outside of the jet maximum

8. Boundary layers, separation, and attachment

Steady Euler’s equation shows that udu/dx ~ -dp/dx; as speed increases pressure decreases

P + ½ rho u^2 = constant along a streamline, so wind slowing to zero on the side of a building

exerts a dynamic pressure

Pressure is highest on the upstream end of an object, lowest on the sides, where the speed

is fastest, and medium in the wake

Fluid entering a constriction moves faster, so pressure decreases downstream

Fluid entering an expansion moves slower, so pressure increases downstream

Irrotational flow impinging on an object introduces vorticity into the flow via viscosity

A boundary layer is characterized by the same sign of vorticity. It grows with the square root

the distance downstream and is inversely proportional to flow speed

Pressure decreasing downstream favors boundary layer attachment;

pressure increasing downstream favors boundary layer growth and separation

airplane wings introduce paired vorticity into flow, related to lift and contrail formation

Sports balls curve when the spin drags the boundary layer with medium pressure to one side, but

low pressure remains on the other side, with a PGF perpendicular to the direction of motion

9. Continuum hypothesis

Below a certain scale successive samples of molecules yield varying

values for speed, temperature, etc., so there is a physical limit

below which fluids really can't be considered continuous.

Collision cross-section

Mean free path

At larger space and time scales there will be variation within a volume,

so we get into the problem of inadequate sampling of real variability,

which undermines forecasting and diagnostic capabilities

The atmosphere and ocean exhibit "red noise" spectra in the sense that

variance tends to get larger at longer space and time scales, a characteristic

aspect of climate and climate change.

To study a global change problem, scales from molecular to global must

be included, with chemical lifetimes varying from less than 1 s to

centuries, another major challenge.

There is an intimate link between smallest and largest scales.

A primary goal is to derive the Navier-Stokes equations.

The gradient of surface stresses gives rise to the Surface Forces on a fluid element. How do we relate the deformation to stress? We need a fourth order tensor to do this.

10. Tensors

Defined by how they rotate

Their order is defined by the number of free indices

Zeroth order - scalars, e.g. Temperature

First order - vectors

Second order - stress tensor (3 directions on each of 3 faces to a cube)

velocity gradient tensor (same)

Kronecker delta

Third order - alternating or permutation tensor, used to define 3D vorticity

Fourth order - 81 element matrix that governs rotation of a second order tensor. Each element could be a nonlinear function of who knows what.

Contraction: set two indices equal to obtain the trace of a matrix

The velocity gradient tensor has symmetric and antisymmetric parts:

symmetric part includes divergence (trace; ignored) and shear strain elements (off-diagonal)

antisymmetric part includes vorticities, which do not deform a fluid element and are ignored in defining the constitutive relationship

11. Newtonian fluid approximation and the Navier-Stokes equations

Since the velocity gradient tensor is the motion of the boundaries of the fluid element, it makes sense that the forces on the fluid element are intimately linked to the velocity gradient tensor. In deriving the Navier-Stokes equations it is assumed that the 4th order tensor that links the two reduces to a constant, while the divergent part of the symmetric part of the velocity gradient tensor is set to zero (incompressibility applied to deriving the viscous stress only).

The normal components of the stress tensor give rise to the pressure gradient force

The tangential components of the stress tensor give rise to the viscous force

Some fluids, like polymer molecules or emulsions (suspended particles) exhibit a nonlinear viscosity with increasing shear or viscoelastic memory. Corn starch in water does not seem to respond linearly with increased shear, rather it becomes more like a solid when stressed.

Using empirical molecular diffusivities is a strength and a blessing for the

Navier-Stokes equations

Net force on a fluid element is given by gravitational acceleration, pressure

gradient force, and viscous force

Assumptions: charge neutral fluid, continuum, incompressibility for viscous force, Newtonian linear coefficient relating shear to stress

Viscous force applies to each velocity component with the 3D Laplacian

Equations are useful for predicting flow evolution at any point in domain, but need to have predictive equation for temperature (hence pressure) to capture the

process of differential heating giving rise to motion through buoyancy.

12. Mass Conservation

Eulerian derivation with fixed volume V and surface S

Using substantive derivative can go back and forth between Eulerian and Lagrangian forms

Lagrangian derivation for fixed mass in a deforming fluid element gives same relation

Density=constant (incompressibility) is a good approximation when the Mach number squared is small. Ma=U/c, where c is the speed of sound. Ma^2 = KE/IE. So incompressibility is a good approximation when wind or wave speeds are much less than the speed of sound. Even so, we need the elemental prime mover of buoyancy production in many problems, so temperature perturbations are retained in the buoyancy term for these situations.

13. Boussinesq approximation

density = constant except in buoyancy term

linear perturbation method

In the vertical momentum equation the imbalance between gravitational and the vertical pressure gradient force is re-written as "reduced gravity" dependent on rho’ or T’in the derivation the quadratic perturbation term and p’ terms are ignored, while the basic state hydrostatic balance is subtracted out

This approximation eliminates sound waves

14. Trace Species Conservation Equation

Gauss' Theorem

Eulerian fixed volume or Lagrangian approach gives

local time rate of change = advection + divergence term + sources/sinks

Even in the absence of sources/sinks, a scalar will not be conserved

following the motion if there are significant vertical displacements.

Volume mixing ratio IS conserved following the motion in the absence

of sources and sinks.

Mass mixing ratio = volume mixing ratio x Molecular weight / Mol. Wt. air

Photochemical time scale = amount / loss processes

Dynamical time scale = length scale / speed scale

The "Chemistry limit": local time rate of change = sources and sinks

(applies when dynamical time scale is very long compared to

chemical time scale; example: OH, which has a lifetime of less than one second)

The "Dynamics limit": local time rate of change = advection

(applies when dynamical time scale is short compared to

the chemical time scale: example: N2O, which lives 180 years)

The most interesting problems are those where chemistry and dynamics

are both important.

AOS models often require simulation of constituents with lifetimes