AOS 610 Geophysical Fluid Dynamics Prof. Hitchman

Problem Set #2 Due Thursday October 26, 2017

1. (1 pt) Let's define an orbit as occurring when $\vec{g}_{eff} = \vec{\nabla}(\Phi_g + \Phi_c) = 0$, where the gravitational and centripetal potential functions are $\Phi_g = GM/r$ and $\Phi_c = \Omega^2 r^2/2$. Solve for the distances r from the center of the earth for an object going around a) once per day and b) once every 28 days.

2. (1.5 pts) Circulation and vorticity.

a) What is the circulation about a circle of radius 500 km for a cyclonic vortex in the Northern Hemisphere, where the tangential wind speed increases outward at 10 m/s per 100 km?

b) What is the mean relative vorticity in the circle?

c) Evaluate the mean vorticity of the earth itself near the north pole, using the same method for a circle of radius 500 km centered at the north pole, assuming that the earth is in solid body rotation and the angular frequency is $\Omega = 2\ \pi\ /\ 1$ day.

3. (1 pt) Thickness.

a) Estimate the 1000-500 hPa thickness for an atmosphere with constant lapse rate 6.5 K/km and T_o=273 K.

b) Examine a 1000-500 hPa thickness chart from a weather analysis or numerical forecast, such as *http* : *//www.aos.wisc.edu/weather/wxmodels/*. What is the mean and the range of thickness in your domain at 45^oN? From this, estimate a typical thickness anomaly.

4. (2 pts) Geostrophic wind and thermal wind.

a) Estimate the geostrophic wind at 4 km over Tateno Japan in Figure 2 of the article about Ooishi and compare with the radiosonde observations in Figure 9 (1^o latitude = 111 km).

b) Surface winds are calm in Madison, but temperatures in the lower troposphere are observed to increase eastward at 1 K per 100 km. Estimate the wind speed and direction over Madison at 5 km altitude if the surface temperature is 290 K and the lapse rate is adiabatic.

5. (1 pt) Curvature versus Coriolis part one.

Consider the horizontal momentum equations on a sphere with no vertical motion. Calculate the curvature and Coriolis terms for a missile fired from Madison toward the northwest at 1000 m/s. After the missile has gone 100 km, estimate how much and in which direction it is deflected from its original path.

6. (1 pt) Curvature versus Coriolis part two. A 1 km radius steel tube through the center of the earth was recently discovered under the city of Madison! a) If you dropped a 1 cm steel ball down the center of it, at what depth would the ball hit the side wall and why? Assume that there is no air friction.

b) Compare what would happen to a dropped ball if the tube through the center of the earth had it's opening at the pole versus at the equator.

7. (1 pt) Tracer continuity equation.

Ozone mixing ratio is observed to be 50 ppbv in Madison and the wind is from the northeast at 20 knots. Stations along the shore of Lake Michigan, 125 km to the east, report ozone observations of 100 ppbv. A photochemical sink of 2 x 10^{-4} ppbv s^{-1} applies throughout Wisconsin. Assuming that ozone only varies in longitude, forecast the ozone mixing ratio 6 hours from now in Madison. (1 m/s $\approx$ 1.94 knots)

8. (1.5 pts) Ideal gas law.

a) Linearize the ideal gas law to show that

$$\frac{P'}{\overline{P}} \approx \frac{\rho'}{\overline{\rho}} + \frac{T'}{\overline{T}} \quad .$$

b) The Boussinesq approximation includes the assumption that $|\frac{P'}{\overline{P}}| << |\frac{T'}{\overline{T}}|$, so that $\frac{\rho'}{\overline{\rho}} \approx -\frac{T'}{\overline{T}}$. By inspecting surface analyses on our website, choose typical values for synoptic scale variations and evaluate the validity of this assumption for the synoptic scale.