AOS 610 Geophysical Fluid Dynamics Prof. Hitchman

Problem Set #3 Due Tuesday, November 21, 2017

Please put your name on your work and staple pages together.

1. (1 pt) Kolmogorov spectrum. Assume that the energy input rate, η, is equal to the rate of dissipation, and equal to the energy transfer rate at all scales k $\sim$ 1/L. Assume also that each eddy transfers its energy in one eddy turnover time $\tau_{eddy} \sim L/U$. Since the total kinetic energy $\sim$ U^2 $\sim$ E(k) k and $\eta \sim U^2/\tau_{eddy}$, show that E(k) $\sim \eta^{2/3}\ k^{-5/3}$.

2. (2 pt) a) Starting from the definition of potential temperature, show that

$$\frac{T}{\theta}\frac{\partial\theta}{\partial z} = \frac{\partial T}{\partial z} + \frac{g}{C_p} = \frac{T}{g}N^2,$$

where N is the buoyancy frequency.

b) Estimate the rate of change of potential temperature with height(in K/km)in the equatorial lower stratosphere.

c) Estimate the buoyancy period, $\tau_b = 2\pi/N$, in the equatorial lower stratosphere.

3. (1 pt) A tropical air mass over the Atlantic near 30^{o}N moves northward conserving absolute vorticity. a) If its initial relative vorticity is zero, what is it at 60^{o}N? b) If the air mass has a 500 km radius, estimate the tangential wind speed near its edge.

4. (2 pts) From Fig. 9.1 in Andrews, Holton and Leovy, estimate Ertel's PV at a) 45^{o}N and 500 hPa and b) 45^{o}N, 100 hPa. Express your answer in "PV units", where 1 PVU = 1 x 10^{-6} K-m^2-kg^{-1}-s^{-1}. (Check your answers against Fig. 9.1b, noting that values in that panel are in units of 1 x 10^{-5} K-m^2-kg^{-1}-s^{-1}.) c) What is the approximate relationship between ozone mixing ratio (panel c) and PV (panel b)? How does it differ above 20 km compared to below 20 km altitude?

5. (2.5 pts.) Consider a well-mixed oceanic boundary layer of depth 20 m near the equator. The wind stress is given by $\tau_{31} = \rho\ c_d\ u_1^2$, where $c_d = .001$.

a) Assuming that there is no friction between the well-mixed layer and the deep ocean, and that all of the energy goes into kinetic energy of the slab upper ocean, how long would it take for the ocean current v to reach the speed of the air, for $u = 5$ m/s?

b) Consider what happens in this formulation when the current speed reaches 5 m/s. How would you modify the relation $\tau = \rho\ c_d\ u^2$, to take into account the current speed v?

c) When the current speed reaches 1 m/s, what is the rate of generation of kinetic energy per unit mass in the slab?

d) Now assume that the effect of the wind stress is distributed downward by turbulence, so that the rate of kinetic energy input into the layer is balanced by the turbulent dissipation rate per unit mass, η. Using your answer to c), calculate the length scale at which deformation work balances molecular diffusion. [The Kolmogorov microscale is $l = \left(\frac{\nu^3}{\eta}\right)^{\frac{1}{4}}$.]

e) Taking a typical value of C_p from Table A3.1 of Gill, what is the rate of change of temperature associated with this kinetic energy input by wind stress?

6. (1.5 pts) Eddy diffusivity.

a) Using eddy mixing length theory, express the acceleration due to vertical momentum flux convergence, $-\frac{\partial}{\partial z}\overline{u'w'}$, in terms of an eddy diffusivity, K_{zz}, and curvature of the time mean flow, $\frac{\partial^2 \overline{u}}{\partial z^2}$. (Turbulence causes departures u' from an average wind $\overline{u}$, so that $u = \overline{u} + u'$. The time mean vertical motion is zero, but turbulence causes vertical eddy winds $w = w'$. In mixing length theory, vertical displacements, δz, cause $u' \approx -\frac{\partial \overline{u}}{\partial z}\delta z$.)

b) If the mean flow profile instead varies in latitude, estimate K_{yy} for "turbulence" associated with synoptic scale systems having characteristic meridional displacements $\delta y \sim 500$ km on a time scale $\delta t \sim 1$ day.