AOS 610 Geophysical Fluid Dynamics Prof. Hitchman

Problem Set #4 10 points Due Thursday December 14, 2017

1. (1 pt) *Deep water waves.* You are going for a walk on the Oregon Coast at 8 am and notice that regular, big waves are breaking on the shore every 18 seconds. At 4 pm later that day waves are breaking every 13 seconds. If these waves were all generated by a wind storm, about how far away was the storm?

2. (1.5 pts) *Orographic gravity waves.* a) Determine the perturbation horizontal and vertical velocity fields and b) evaluate the vertical flux of westerly momentum $\rho\ \overline{u'w'}$ (Pa) for stationary gravity waves forced by westerly flow over sinusoidally-varying topography $h = h_o\ cos\ k\ x$, where $h_o = 100$ m, $N = 2 \text{ x } 10^{-2}\ \text{s}^{-1}$, $\overline{u} = 5$ m/s, and $L_x = 5$ km. We can approximate the lower boundary condition as $w'(0) = dh/dt = \overline{u}\ \frac{\partial h}{\partial x}$.

3. (2 pts) *Two layer model of tropical Pacific Ocean.* The water surface is at height η above the bottom of the ocean and there is a thermocline at height h above the bottom of the ocean. Let the upper layer be warmer than the lower layer by 10 K and have a depth of 200 m, with a total depth of 4 km.

a) Find the baroclinic equivalent depth and corresponding phase speed (cf. Gill 6.2.22).

b) For the baroclinic mode, what is the ratio $|\eta|/|h|$ (cf. Gill 6.2.13)?

c) An oceanic Kelvin wave trapped at the equator can be approximated by an eastward-travelling baroclinic mode. For an oceanic Kelvin wave, estimate the equatorial Rossby radius of deformation $(c/2\beta)^{1/2}$ from your result in a).

d) About how long would it take a Kelvin wave to travel along the equator from New Guinea to Peru?

4. (1 pt) *Rossby radii of deformation.* a) Given that low pressure centers for midlatitude synoptic scale cyclones tilt westward with altitude, such that the trough near 500 hPa ($\sim$5 km) overlies the surface ridge, estimate the baroclinic Rossby radius of deformation for these waves, $L_R = N/|mf|$.

b) What do you think is the most fundamental reason that L_R is $\sim$10 times bigger in the atmosphere than in the ocean?

5. (2 pts) *How accurate is the Rossby wave dispersion relation?* Pick a wave feature of interest, track its eastward progress for a few days, and compare it with theory. Go to http://www.aos.wisc.edu/weather/Models to view NH polar stereographic projections of weather forecasts and look for troughs and ridges in the white geopotential height contours at 500 or 300 or 250 hPa.

a) Estimate the observed zonal trace speed C_{obs} of a chosen midlatitude trough or ridge from a sequence of synoptic charts. Print out and attach sample charts and indicate your chosen trough and ridge axes. Estimate how fast they travelled along a latitude circle during your observation time, δt, using $C_{obs} \approx \delta x / \delta t$, where $\delta x = a \cos \phi \, \delta\lambda * \pi/180$, with $\delta\lambda$ in degrees longitude. You may need to consult a globe to find latitude and longitude.

b) Compare your observed estimate with an estimate from the theoretical zonal trace speed, c_t, obtained from the linear barotropic dispersion relation. You will need to estimate the zonal and meridional wavelengths (k and l), assuming an infinite vertical wavelength (m = 0). You will also need to estimate the average zonal wind, $\overline{u}$, throughout the region where your wave exists, and choose a central latitude for estimating β and $cos\phi$ from an auxilliary map.

6. (2.5 pts) *Rossby-type waves in a rotating tank.* The angular frequency is Ω and the fluid depth decreases toward the center as $H(y) = H_o - \gamma\, y$, due to a sloping bottom. [In this tank y increases toward the rotation axis and z increases downward, so $v > 0$ is toward the center, but $w > 0$ is downward!] Assuming that the velocity field is geostrophic except in the divergence term,

a) show that the perturbation continuity equation can be written

$$H_o \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) - \gamma v' = 0$$

and

b) that the perturbation potential vorticity equation is

$$\frac{\partial}{\partial t} \nabla^2 \psi' + \beta \frac{\partial \psi'}{\partial x} = 0$$

and $\beta = 2\Omega\gamma/H_o$. Thus, the sloping bottom gives an "equivalent beta effect".

c) What is the Rossby wave phase speed for waves of wavelength 50 cm in the azimuthal (local Cartesian x direction) if $\Omega = 1\ \text{s}^{-1}$, $H_o = 20$ cm, and $\gamma = 0.1$?