Wave-CISK and Convective Mesosystems

DAVID J. RAYMOND

New Mexico Institute of Mining and Technology, Socorro 87801

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ABSTRACT

With the CISK hypothesis (Convective Instability of the Second Kind), Charney and Eliassen (1964) and Ooyama (1964) introduced a simple means of handling the interaction between cumulus convection and a larger scale flow. The author has shown that a wave-CISK model (i.e., the larger scale flow resembles a gravity wave) can be used to predict the motion of severe convective storms.

The purpose of this paper is to show that the above model explains the structure and rapid development of these disturbances. It is found that flow patterns characteristic of such systems can evolve explosively from an initially weak, structureless region of low-level convergence in less than 1 h.

The present model does not include the effects of environmental rotation. This limitation apparently prevents the model from properly simulating the tornado cyclone.

1. Introduction

The characteristics of convective mesosystems have been extensively documented by Fujita (1963). These systems appear to originate as a localized region of convection, subsequently spreading out like a ripple on a pond. However, unlike water waves, the convective wave rarely spreads in symmetric fashion—phase speed and convective intensity can vary greatly with azimuth, and convection may be entirely absent from certain sectors. Thus an expanding arc of convection is more often observed than a full circle. This arc is frequently interpreted as a squall line.

Convective mesosystems generally develop to maturity in one hour or less. At this point wind and pressure perturbations reach their maximum intensity. These maxima can be maintained for many hours, during which time the mesosystem increases in area. The most intense convergence and the strongest convection is found along the expanding edge of the mesosystem, while the interior becomes filled with cool, convectively stable air. The mesosystem presumably dies when it uses up all of the unstable air available to it. Lifetimes can range from a few hours to more than a day, and maximum dimensions of 1000 km or more have been observed.

Any model purporting to simulate a convective mesosystem must deal with motions on a wide range of scales. Vertical transport takes place primarily in convective up- and downdrafts perhaps 5 km in diameter. On the other hand, horizontal transport depends upon motions similar in scale to the width of the convective arc, perhaps 50–100 km. Furthermore, the arc itself may be as long as 500 km. Numerical models are as yet unable to encompass a horizontal scale of 500 km (or even 100 km!) and still directly simulate convective scale motions. We therefore must calculate the bulk statistical effects of the convective scale in our models, and conversely, must discover how the convective scale motions respond to mesoscale promptings. The former problem yields to a straightforward application of existing cloud models, and numerous investigators (see for instance, Arakawa and Schubert, 1974) have pursued this track. The latter problem continues to defy analysis, and possibly ranks as the most important unsolved problem in atmospheric dynamics. However, the CISK model of Charney and Eliassen (1964) and Ooyama (1964) provides a plausible, if only partial solution to this difficulty.

Stated briefly, the CISK hypothesis postulates that convection will process just that amount of air brought to the condensation level (or the level of free convection) by boundary layer motions. Charney and Eliassen hypothesized frictionally induced convergence to be the dominant boundary layer supply mechanism. However, Lindzen (1974) showed that gravity waves are also efficient producers of low-level convergence, and moreover can respond positively to the heat flux associated with the resulting convection. The resulting instability was dubbed “wave-CISK” by Lindzen.

The author (Raymond, 1975; referred to hereafter as R), has shown how to calculate the properties of wave-CISK modes for real atmospheric profiles of wind and temperature. The group velocities of the most unstable wave-CISK modes were often found to correspond with the propagation velocities of severe
convective storms. Wave-CISK thus becomes a plausible basis for a severe storm model.

The purpose of this paper is to show how the wave-CISK model can explain the structure and explosive development of the convective mesosystem. The theory of R will be extended to the point where the form and evolution of flow patterns may be calculated. The method is basically to resolve an assumed initial flow pattern into wave-CISK eigenmodes, let each eigenmode evolve independently (as is allowed by linear theory), and then recombine the evolved eigenmodes to obtain the new flow pattern. Two case studies will then be presented for which the computed flows evolve from simple, nonstratified initial conditions into patterns resembling those actually observed. Since the model is linear, it is strictly valid only for the initial growth phase of a mesosystem. However, since the growth phase is perhaps the most poorly understood, a linear model is still of considerable value.

2. Theory

We wish to use R's model to calculate the evolution of wave-CISK flow patterns. Given an assumed exponential dependence of all fields on the horizontal space \((x,y)\) and time \(t\) variables of the form \(\exp[\ii (k_x x + k_y y - \omega t)]\), a method for calculating \(\omega\) and a function \(\psi'(z)\) was developed by R. The vertical profiles of all fields was related to \(\psi'.\) For shallow convection, \(\psi'\) reduces to the smoothed mesoscale vertical velocity, \(\langle v_z'\rangle.\) When the effects of scale height are significant, \(\psi'\) differs slightly from \(\langle v_z'\rangle:\)

\[
\langle v'_z \rangle = \psi \exp[(2 - 1/\gamma)\mu^* z/2],
\]

where \(\mu^* = 1\) is the atmospheric scale height (assumed constant and \(\approx 8\) km) and \(\gamma = 1.4\) is the ratio of specific heats for dry air.

Defining \(k = k_x \hat{x} + k_y \hat{y}\) and \(n = -k_y \hat{x} + k_x \hat{y},\) we shall use a subscript \(k, n\) or \(z\) to indicate the component of a vector in the associated direction. The anelastic continuity equation may be written for the mesoscale wind perturbation \(\psi'\) as

\[
\frac{\partial \langle v'_z \rangle}{\partial z} + \frac{\mu^*}{ik} \langle v'_z \rangle = 0
\]

where \(k = |k|\). From (2), \(\langle v'_z \rangle\) is found to have the form

\[
\langle v'_z \rangle = \frac{1}{ik} \left[ \frac{\partial \langle v'_z \rangle}{\partial z} + \mu^* \langle v'_z \rangle \right].
\]

For the purposes of R, it was not necessary to compute \(\langle v'_z \rangle\), the horizontal wind perturbation normal to \(k,\) since this component does not enter into the calculation of \(\omega.\) However, it is easily shown that \(\langle v'_z \rangle\) obeys the equation

\[
-i \omega \langle v'_z \rangle + ik V_k \langle v'_z \rangle'' + \langle v'_z \rangle'' = -\langle v' \cdot \nabla v'_z \rangle,
\]

where \(V(z)\) is the ambient wind profile. We shall ignore the term on the right side of (4) which describes the convective transport of the \(n\) component of momentum. This is consistent with the omission of the corresponding \(k\) component by R. We may thus solve (4) for \(\langle v'_z \rangle:\)

\[
\langle v'_z \rangle = \frac{\langle v'_z \rangle''}{ik(c - V_k)} dz.
\]

Use of the relations

\[
\langle v'_x \rangle = \hat{x} \cdot \hat{k} \langle v'_z \rangle + \hat{x} \cdot \hat{n} \langle v'_z \rangle,''
\]

\[
\langle v'_y \rangle = \hat{y} \cdot \hat{k} \langle v'_z \rangle + \hat{y} \cdot \hat{n} \langle v'_z \rangle,''
\]

completes the calculation of the Cartesian components of the mesoscale wind perturbation.

Recall that \(\langle \psi'(k,z) \rangle\) is the eigenfunction associated with a particular wave vector \(k.\) Any possible wave-CISK flow pattern \(\psi(r,t)\) may be constructed from an appropriately weighted superposition of these eigenfunctions, plus the ambient flow:

\[
\psi(r,t) = \sum \int dx dy C(k) \psi'(k,z) e^{i(k \cdot r - \omega t)}
\]

Here \(V(z)\) is the ambient wind profile and \(C(k)\) a weighting function that depends upon the initial conditions. We hypothesize an initial distribution of surface convergence of the form

\[
x(x,y) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \bigg|_{x,t=0}
\]

Applying (8), we find that (9) reduces to

\[
x(x,y) = \sum \int dx \int dy C(k) \langle -i k v'_x(k,0) \rangle e^{i k \cdot r}.
\]

(10)

which is simply a double Fourier transform. Since eigenfunctions are determined only up to a constant multiplier, we may normalize \(\langle \psi' \rangle\) so that

\[
\langle -i k v'_x(k,0) \rangle = 1.
\]

(11)

Inversion of the transform completes the evaluation of \(C:\)

\[
C(k) = (2\pi)^{-1} \int dx \int dy x(x,y) e^{-i k \cdot r}.
\]

(12)

Using (8) in conjunction with (12), we may calculate the time development and vertical structure of any wave-CISK disturbance given the initial distribution of surface convergence.

The phase velocity \(c(k)\) was evaluated only in the limit \(k = 0\) by R. This was justified by a scale analysis which suggested that the effect of realistic finite wavelengths would be negligible. In actuality, rather more
dispersion occurs than is suggested by scale analysis. For this reason the long wavelength approximation was not applied in this paper.

3. Grover storm of 22 July 1972

Two case studies will be presented in which a weak, azimuthally symmetric region of surface convergence of the form

$$X(x,y) = X_0 e^{-(x^2+y^2)/R^2} \quad (13)$$

evolves into a complex, asymmetric flow pattern comparable in structure to the observed convective meso-

system. This computed structure results from dynamics dependent primarily upon the environmental sounding—varying the size and strength of the initial convergent region has only minimal effect.

Our first example will be the Grover, Colo., storm of 22 July 1972. Foote and Fankhauser (1973, referred to hereafter as FF) documented the surface flow pattern associated with this storm as it passed over the National Hall Research Experiment (NHRE) network.

Fig. 1 shows the real and imaginary parts of the phase velocity, as calculated by the method of R, for the Grover storm environment. The peak in $\text{Im}(c)$ varies from 120° for $k = 0 \text{ km}^{-1}$ to 180° for $k = 0.5 \text{ km}^{-1}$, which suggests that the eigenmode will have a principal wave vector oriented in this range.

Fig. 2 shows the variation of $c$ with $k$ for $\phi = 150^\circ$. The strong decrease of $\text{Im}(c)$ with $k$ occurs in most instances studied by the author, and as is also shown in Fig. 2, this results in a maximum in $\text{Im}(c)$ for finite $k$. The theory thus predicts a preferred finite wavelength for the mode of maximum growth rate. This wavelength is typically six times the height of cloud base AGL.

To conserve computer time, $c(\phi)$ was in general calculated only for 0 and 0.5 km$^{-1}$. Other values were obtained by linear interpolation. As the dashed straight line approximations in Fig. 2 show, the use of linear interpolation in $k$ results in a modest amount of error. The small fluctuation that occurs near $k = 0.2 \text{ km}^{-1}$ is possibly due to some type of resonance. Such fluctuations occur occasionally over limited azimuthal ranges. In this instance, it is not present for azimuths 10° to either side of 150°.

Eq. (12) was approximated by the sum

$$C(k, x_0) = (2\pi)^{-2} \sum_{m, n=1}^{19} X(x_m, y_n) e^{-i(kx_m + ky_n)} (\Delta x)^2, \quad (14)$$

where $\Delta x = 5 \text{ km}$. The domain of integration was thus restricted to a region 95 km square. Similarly, (8) was reduced to a sum over a 19 by 19 square grid centered on the origin with sides equal to $2\pi/(19\Delta x)$. These approximations force periodic boundary conditions with a 95 km period. This restriction is not deemed serious for the growth stage of a convective meso-system when it is still quite limited in area. It is clear that better computational technique would be required during the mature stages.

a. Evolution of surface flow

Fig. 3 shows the calculated evolution of the flow pattern, starting with $x_0 = 5 \times 10^4 \text{ s}^{-1}$ and $R = 7.5 \text{ km}$ in (13). Note how the convergence pattern grows in intensity and develops a region of divergence as it moves eastward. The flow responds accordingly, with the development of a curved confluen line to the
storm dimensions would be considerably greater, but storm amplitude would have grown to an impossibly large value. We conclude that by 1655 the Grover storm had indeed reached the stage at which nonlinear effects precluded further intensification. The actual and calculated patterns are therefore not strictly comparable. Nevertheless, the observed degree of similarity is encouraging.

b. Storm-relative flow

A better understanding of storm flow patterns may be had by evaluating the flow relative to the moving storm. Fig. 4 shows the storm-relative flow at $t = 1400$ s for elevations of 0 and 1.5 km. The hatched regions for $z = 0$ km indicate where convergence or divergence exceeds $2 \times 10^{-4} \text{ s}^{-1}$. The general appearance of the computed $z = 0$ km flow is similar to the actual storm-relative flow, shown in FF’s Fig. 11b. FF also show, in their Figs. 15 and 16, somewhat speculative plots of upper level wind fluctuations from aircraft measurements. The flow at $z = 1.5$ km was calculated for comparison with these measurements. The dotted line

Fig. 3. Simulated evolution of surface flow for Grover storm of 22 July 1972. Hatched areas indicate regions of convergence exceeding the labeled value, which is given in units of $10^{-4}$ s$^{-1}$. Divergence is indicated by a minus sign. The track of the storm is plotted in the diagram on the lower right, along with computed and (in parentheses) observed velocities. North is up, and distances are in kilometers.

The primary difference between the observed and calculated patterns is in the size of the disturbance. At 1655 MST, the Grover storm was at least 50 km across in the N–S direction, while at 1200 s the simulated storm was closer to 25 km in diameter. This is related to the relative ages of the actual and simulated storms. At 1655, the Grover storm had been moving to the right of the mean wind for nearly 1 h, or 3 times the age of the simulated storm. If the model calculation were allowed to progress for 1 h, the simulated

Fig. 4. Computed flow relative to Grover storm at $t = 1400$ s. The hatched areas in the $z = 0$ flow indicate convergence as in Fig. 3, while hatching indicates vertical winds in excess of the labeled value (m s$^{-1}$) for $z = 1.5$ km. The dotted line is a simulated aircraft track bearing the same relation to the computer storm that real track bore to the actual storm (see Foote and Fankhauser, 1973, Figs. 15 and 16). The short line segments attached to the aircraft track show simulated wind vectors along the track for purposes of comparison with Foote and Fankhauser’s data. A vector length equivalent to the grid spacing indicates 30 m s$^{-1}$.
4. The storms of 3 April 1964

On 3 April 1964 a remarkable series of storms occurred in central Oklahoma. (See Charba and Sasaki, 1971.) These storms multiplied from a single ancestor by the splitting process, in which a parent storm divides into two parts, one of which moves to the left of the mean wind, and the other to the right. Splits occurred several times, subjecting a large area to severe weather. As this system is particularly illustrative of splitting, it was chosen for our second case study.

a. Evolution of surface flow

Fig. 5 illustrates the development of the surface flow associated with environmental conditions present on 3 April 1964. The same initial convergence pattern as for the Grover storm was used. As may be seen, this convergent region first expands, and then splits into two parts, with a divergent region forming in between. This divergent region also splits, and the two separate convergence-divergence couplets move off on their own. The left couplet moves with a velocity of 17.5 m s\(^{-1}\) from 178°, while the right one propagates more slowly at 11.2 m s\(^{-1}\) from 257°. This compares with the observed velocities of left and right moving storms respectively of 14 m s\(^{-1}\) from 187° and 10 m s\(^{-1}\) from 250°, indicating excellent agreement between theory and observation. By t = 2000 s the theory predicts convergence in excess of 10\(^{-2}\) s\(^{-1}\) for the right-moving storm, which is clearly excessive. This, and the development of strong convection in old downdraft air between the left and right moving storms, indicates that by this time the limit of validity has been reached for a linear theory.

Fig. 6 shows the computed \(\text{Im}(c)\) as a function of \(\phi\) for the conditions of 3 April 1964. The reason for the splitting behavior is quite clear: \(\text{Im}(c)\) is bimodal, with peaks at azimuths of 150° and 315°. This leads to the formation of two unstable wave packets, each associated with one of the peaks in \(\text{Im}(c)\). For most situations one finds that \(\text{Im}(c)\) has only one peak in azimuth, and only one wave packet is thus created. This corresponds to the observed fact that split pairs with both components being severe are rare.

b. Vertical structure

In this section the flow aloft is discussed. We wish to infer the nearly steady flow pattern associated with
a mature storm. For this reason, the calculated flow is displayed for the time at which the maximum surface convergence has grown to $5 \times 10^{-4}$ s$^{-1}$. This implies an updraft of 10 m s$^{-1}$ at 2 km AGL, which from the observations of Marwitz (1972) is typical of a severe supercell storm. The left and right components are treated separately in these calculations by restricting the integration in (8) to an azimuthal range including only the appropriate peak in $\text{Im}(c)$. The resulting flow patterns, shown in Fig. 7, also differ from Fig. 5 in that the streamlines are storm-relative.

Note that the updraft region is on the NW flank of the left mover and the SE flank of the right mover. The surface flow of the right mover is not completely characteristic of other cases studied by the author, in that the inflowing low level air appears to impinge on both sides of the updraft. For the Geary storm of Browning and Donaldson (1963) and the supercell storm of Marwitz (1972), the inflowing air has a more southerly component, entering the updraft only on the SE side. The low level flows of the left and right components in this case are quite close to being mirror images of each other.

The curvature in the streamlines at 3 km shows the strong effect of the tilting term in inducing a vertical vorticity component. Vorticity up to roughly $5 \times 10^{-3}$ s$^{-1}$ is induced at this level in the updraft of the right mover, while the left mover exhibits a maximum anticyclonic vorticity of $-9\times10^{-3}$ s$^{-1}$ in its updraft region. Vorticity computed in downdrafts is comparable in magnitude and opposite in sign to that in the updraft of the associated storm.

At 6 km the flow outside the storm resembles that produced by the blocking effect of a solid obstacle. However, the resemblance is only superficial, as the storm is intimately coupled to the environmental flow at this level.

c. Speculations concerning tornado formation

With minor exceptions, the computed flow patterns for the left- and right-moving storms of 3 April 1964 are mirror images of each other. This symmetry is also obvious in the relationship between the wind hodograph and storm propagation velocities as shown in Fig. 8. Except for the location of the origin, Fig. 8 is nearly symmetric with respect to reflection about the line AB—the only major result of this operation is to interchange the left and right moving components.

The above result has important implications for tornado production. Only two possible sources of vorticity exist for the formation of tornadoes—the previously discussed tilting term and environmental absolute vorticity which may be concentrated by convergence. The present model contains the former mechanism, but not the latter, since the environment is assumed not to rotate.

![Wind hodograph plus computed (diamond) and observed (circle) velocities of Oklahoma storms. The line AB is nearly a line of reflective symmetry, which indicates that left (L) and right (R) moving storms should be approximate mirror images of each other.](image)
If tilting alone were responsible for tornado production, then the above mentioned left-right symmetry means that the left and right movers should have produced anticyclonic and cyclonic tornadoes respectively. However, no tornadoes were observed from the left components in this case even though they attained extreme severity. On the other hand, numerous tornadoes and funnel clouds were produced by the right movers. Since environmental absolute vorticity is generally cyclonic, we conclude that its concentration in these storms was instrumental in suppressing rotation in the left components, and enhancing it in the right components. A reasonably convincing argument is thus made for the importance of environmental vorticity to tornado production.

5. Discussion

The wave-CISK model has been shown to provide an excellent simulation of the structure and movement of the two illustrated convective mesosystems. In addition, the rapid development commonly observed in such systems is well represented.

Unfortunately, it is difficult to extract a plausible physical explanation for the movement of severe storms from the mathematical model. However, certain empirical rules emerge from examining the results of numerous case studies. First, if the wind profile is precisely planar (i.e., the hodograph is a straight line), no deviate motion occurs, and the storm moves with the wind at some level. However, if the hodograph is curved, especially in the lowest few kilometers, the storm motion will tend to fall on the concave side of the curve. Since hodographs of severe storm environments tend to curve to the right in the lowest 4 to 5 km, the commonly observed, rightward deviation of severe storms is thus explained. If no pronounced curvature of the hodograph is present, but the wind profile is still not exactly planar, splitting behavior is possible. However, whether the necessary bimodal $\text{Im}(c)$ structure actually occurs is a sensitive function of the details of the environmental sounding, and no general conclusion may be stated.

The production of vorticity aloft via the tilting term is an inevitable result of deviate storm motion in a sheared environment. Thus, rotational tendencies aloft are probably common in severe storms. However, as the 3 April 1964 case study shows, this vorticity from tilting cannot, by itself, explain tornado production. It would appear that collaboration between tilting and concentration of environmental vorticity is necessary. Extension of the wave-CISK model to rotating fluids is clearly an important step.

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REFERENCES


