A Wave-CISK Model of Squall Lines

DAVID J. RAYMOND

Department of Physics and Geophysical Research Center, New Mexico Institute of Mining and Technology, Socorro, NM 87801

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ABSTRACT

The wave-CISK model of Raymond incorporating lag effects in the updraft and downdraft is implemented as an initial value problem in physical space. Examples of both midlatitude and tropical squall lines are successfully simulated. Diagnosis of the model shows that condensational heating in the updraft is the primary driving mechanism of a mature squall line, with evaporative cooling and convective momentum transfer playing subsidiary roles. However, an instability involving the displacement of boundary layer air by downdrafts apparently plays an important role in squall line initiation.

A weakness of the model is its inability to predict the direction of squall line propagation relative to the low level wind shear. This is traced to the insensitivity of the convective parameterization to midlevel entrainment. However, unlike strict two-dimensional squall line models, the parameterization allows cross-stream mass transfer to occur in the context of overall slab symmetry. Such transfer is dynamically important for at least some squall lines.

1. Introduction

Squall lines are a particularly inviting target for numerical simulation because of their apparent two dimensionality. A number of investigators have constructed two-dimensional squall line models, e.g., Takeda (1971), Schlesinger (1973), Hane (1973), Moncrieff and Green (1972), Thorpe et al. (1982) and Seitter and Kuo (1983).

A problem with such models has been the difficulty of reproducing the observed upshear slope of squall line updrafts (Moncrieff, 1978). Recently, ways around this problem have been found. Thorpe et al. (1982) demonstrated that concentrating the shear in the lowest few kilometers results in the required upshear slope in both a steady-state model and a time-dependent numerical simulation. Seitter and Kuo (1983) showed that precipitation loading and evaporation can also cause upshear sloping updrafts provided that these effects are concentrated in the lower part of the cloud. This distinguishes their results from those of Moncrieff (1978) previously mentioned, which incorporate a downdraft that is uniformly forced with height.

It appears, therefore, that a dynamically consistent model of steady, two-dimensional moist convection is possible. However, other evidence indicates that at least some squall lines are essentially three dimensional. In particular, many squall lines have the updraft and downdraft both feeding from the same side (i.e., the "front") of the storm (Zipser, 1969, 1977; Houze, 1977; Ogura and Liou, 1980). This implies a crossing of updraft and downdraft flows in a vertical plane normal to the squall line and consequent violation of strict two dimensionality. Such a picture contrasts with the classic view of Browning and Ludlam (1962) which has downdraft air entering from the rear.

Conservation laws also argue against a steady, two dimensional interpretation of mean streamline patterns such as those presented by Newton (1950) and Ogura and Liou (1980). Careful examination shows that equivalent potential temperature is generally not constant along streamlines, suggesting either significant cross-stream mass exchange or strongly time-dependent flow.

Further examination of observations occasionally shows the updraft to have slopes of 0.1–0.2. It seems unlikely that conditionally unstable air could ascend on such a shallow slope without undergoing immediate overturning once the level of free convection is reached. Indeed, the models typically exhibit much steeper slopes.

In summary, squall line observations in which two dimensionality and strong time smoothing are enforced by the analysis may prove to be a trap for the unwary modeler; the dynamics of two dimensional steady flow impose stringent conditions on the solution which may not be satisfied by real squall lines.

The only three-dimensional models of squall lines to date are those of Moncrieff and Miller (1976). Their analytical model allows updraft and downdraft streamlines to cross while retaining the two-dimensionality of a squall line in an average sense. This model makes a prediction of squall line propagation speed that is in rough accord with observation. Moncrieff and Miller's (1976) numerical model produces a linear array of convective cells that on the whole
looks like a tropical squall line. It is interesting that the simulated squall line arises not from the initial convective cell, but indirectly as a result of the cold air outflow created by the decay of this cell.

The picture of a squall line as a linear array of convective cells suggests a model in which three-dimensionality is confined to a convective parameterization. In this manner, mass transfer could occur across two-dimensional streamlines, which would be considered only indicators of the average motion rather than actual parcel paths.

Ooyama (1982) has pointed out some of the hazards involved in parameterizing convection. In particular, if latent heat release is included in the explicit part of the calculation, one runs the chance of making a distorted explicit calculation of the effects of convection. This in conjunction with the parameterization could result in double counting of convective effects. A related difficulty is that primitive equation models which include a convective parameterization sometimes generate disturbances with the same time and space scales as the convection itself. This invalidates parameterization schemes that are dependent upon a scale separation to delineate what is parameterized and what is explicitly computed.

Curiously, the wave-CISK model, which was singled out by Ooyama (1982) as embodying the above ills, does not suffer from the first and can at least be partially cured of the second. In particular, the boundary between parameterized and explicitly computed processes in wave-CISK is defined by thermodynamics rather than scale; all moist processes are parameterized, leaving only dry processes to explicit computation. Thus, no cloud can form without the aid of the parameterization.

Davies (1979) and Raymond (1983) pointed out a possible solution to the second ill. Convection in wave-CISK is initiated by lifting of unstable boundary layer air above the level of free convection. In conventional formulations of wave-CISK (e.g., Lindzen, 1974; Raymond, 1975, 1976), this convection is presumed to act instantaneously on the explicitly computed flow. However, appropriately lagging the release of latent heat and subsequent evaporative cooling can provide an approximation to the complex, time-dependent processes that actually occur in convective cells.

Ooyama (1982) also raised the philosophical question as to whether wave-CISK is simply “discussing convective clouds in disguise.” It is a matter of interpretation as to whether a squall line is simply an overgrown convective cloud or a structure discernible from the convective cells that constitute it. In the latter case, wave-CISK is to squall lines as frictional CISK is to tropical cyclones; only the mechanism by which the higher level structure creates the convection is different. If, on the other hand, wave-CISK is simply a model for a type of convective cloud that contains many cells and is self-exciting, then perhaps the acronym is poorly chosen. However, this does not destroy it as a potentially useful description of a real phenomenon.

This paper is an extension to the wave-CISK model of Raymond (1983), hereinafter denoted R, where downdraft and lag effects were treated via an eigenmode analysis. Three extensions to this model are made here:

1) The calculations are performed as initial value problems in physical space. This has the benefit of making the results more accessible to intuition.

2) A momentum transfer parameterization similar to that of Ooyama (1971) is included for the purpose of testing Newton’s (1950) hypothesis of squall line propagation.

3) A moist static energy budget is included so that convection can be shut off when boundary layer instability is exhausted.

The plan of the paper is as follows: In Section 2 the linearized, hydrostatic, nonrotating Boussinesq equations of R are introduced, and the cumulus parameterization developed. Section 3 covers the somewhat novel mathematical approach to the solution of the equations. Two squall lines, a tropical and a midlatitude case, are simulated in Section 4. Section 5 discusses various diagnostic tests that reveal how the simulated storms function. Finally, the results and their implications are discussed in Section 6.

2. Parameterization

In two dimensions the equations used by R follow:

\[
\frac{\partial v}{\partial t} + V \frac{\partial v}{\partial x} + \frac{dV}{dz} w + \frac{\partial \pi}{\partial x} = F, \tag{1}
\]

\[
\frac{\partial \eta}{\partial z} = 0, \tag{2}
\]

\[
\frac{\partial \eta}{\partial t} + V \frac{\partial \eta}{\partial x} + N^2 w = H, \tag{3}
\]

\[
\frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} = 0. \tag{4}
\]

The variables \( v \) and \( w \) are, respectively, the horizontal and vertical velocity perturbations; \( \pi \) is the pressure perturbation divided by the density, and \( \eta \) the acceleration of gravity times the fractional virtual potential temperature perturbation, denoting the buoyancy; \( V(z) \) and \( N(z) \) are, respectively, the vertical profiles of ambient horizontal wind and Brunt-Väisälä frequency; \( H \) is the convective heat source, and is equivalent to the \( Q_z \) of the diagnostic models. (See, for instance, Yanai et al., 1973.; \( F = -\partial(v^2)/\partial x - \partial(vw)/\partial z \approx -\partial(vw)/\partial z \) is minus the divergence of the perturbation momentum flux.)
The moist static energy perturbation $\sigma$ and its ambient profile $S(z)$ satisfy
\[ \frac{\partial \sigma}{\partial t} + V \frac{\partial \sigma}{\partial x} + \frac{dS}{dz} w = Q. \] (5)

Analogous to $F$ in the momentum equation, $Q$ is minus the divergence of convective moist static energy flux.

It was pointed out by R that the cumulus mass flux $M$ is approximately related to convective heat source $H$ by the formula
\[ H = N^2 M. \] (6)

Physically, this is equivalent to saying that air detrained from a cloud comes out with the density of the environment at the exit level. Mathematically it can be derived by averaging (3) over a horizontal slice through a cloud, ignoring space and time variations in $n$, and identifying $w$ with $M$.

Equation (6) breaks down in a neutrally stratified boundary layer. Thus, evaporative cooling below cloud base is not well represented. A crude method of accounting for this will be discussed later.

Using (6), we note that cumulus heating can be immediately related to boundary layer processes that determine the cumulus mass flux. In particular, we equate the cumulus mass flux at the level of free convection (LFC) $z = b$ to the computed vertical velocity at that level.

When moist downdrafts are present, separate matching conditions for updrafts and downdrafts are defined. Dividing $M$ into updraft and downdraft components, we obtain
\[ M = M_u + M_d. \] (7)

Let
\[ M_u(b) = w_u, \] (8)
\[ M_d(b) \geq w_d, \] (9)
where
\[ w_u(x, b, \tau) = \max[w(x, b, \tau), 0], \] (10)
\[ w_d(x, b, \tau) = \min[w(x, b, \tau), 0]. \] (11)

Adding (10) and (11) we get
\[ w(x, b, \tau) = w_u(x, b, \tau) + w_d(x, b, \tau). \] (12)

The reasoning behind (8) and (9) is as follows: Any upward motion of unstable, boundary layer air through the LFC results in the formation of a cumulus cloud and associated cumulus mass flux. Hence the equality in (8). However, there is no similar requirement for downdrafts—these can be either moist or dry, depending on the situation. The condition on $w_d$ is an inequality that ensures only that the dry downward motion $w_d - M_d$ is truly downward. (Note that $M_d$ and $w_d$ are both negative by definition.)

It is clear that $w_u$ is the key element in initiating convection. After Ooyama (1971), $w_u$ is called the dispatcher function. Combining (9) and (12) results in
\[ w_u \geq w(b) - M_d(b) \] (13)
which has the following interpretation when the equality holds: A nonzero dispatcher function results from a combination of mean upward motion and displacement of boundary layer air by moist downdrafts. The inequality permits the additional possibility that dry downdrafts displace boundary layer air as well. In this case $w_u$ exceeds the value it would take if only moist downdrafts entered the boundary layer.

We make the conservative assumption that dry downdrafts have negligible effect in forcing new updrafts and replace (13) by
\[ w_u = \max[w(b) - M_d(b), 0], \] (14)
where account has been taken of the fact that the dispatcher function cannot be negative.

Raymond (1983) showed how to relate the convective mass flux to the dispatcher function in the case where lags occur:
\[ M(x, z, \tau) = \frac{1}{b} \int_0^\infty D(z, \tau) w_u(x - V_d \tau, t - \tau) d\tau, \] (15)
where $D$ is called the lagged mass flux, representing the vertical profile of cumulus mass flux a time $\tau$ after the impulsive initiation of convection; $V_d$ is the advection velocity of evolving convective cells, and assumed here to be constant.

Corresponding to the partition of $M$, $D$ is divided into updraft and downdraft parts:
\[ D = D_u + D_d. \] (16)

Satisfaction of (8) requires that
\[ D_u(b, \tau) = b \delta(\tau), \] (17)
An assumption of the foregoing scheme is that the boundary layer always consists of unstable air. Moist downdrafts clearly can modify the boundary layer to an extent that this assumption is no longer valid. We therefore impose a somewhat arbitrary further condition: if
\[ \frac{-\sigma(b)}{S(b)} > 0.1, \] (18)
i.e., if the depletion of the moist static energy at the LFC exceeds 10%, the dispatcher function is turned off at that location.

An additional point needs clarification. Simultaneous updrafts and downdrafts at precisely the same time and place are not possible. Thus, $w_u > 0$ implies that $M_d = 0$ at that point. Strictly speaking, it is therefore not necessary to include $M_d$ in (14), since $w_u = 0$ when $M_d = 0$. However computer limitations force finite resolution on any model, and it may not be possible to resolve explicitly each updraft and downdraft. In this case an implicit smoothing is imposed by the computational grid size, causing adjacent updrafts and downdrafts to appear to overlap. This possibility requires that $M_d$ be retained in (14). The
contributions of $w$ and $M_d$ to the dispatcher function are, respectively, denoted convergence forcing and displacement forcing. However, it should be clearly understood that displacement forcing is simply unresolved convergence forcing. 

Turn now to a specific form for $D(z, \tau)$: imagine a bubble rising at a constant speed $W$. Then $D_u$ takes the assumed form

$$D_u(z, \tau) = \frac{P_u(z) \delta[\tau - (z - b)/W]}{1}.$$  \hspace{1cm} (19)

This is consistent with (17) if $P_u(b) = b$.

Further, imagine that precipitation impulsively begins when the rising bubble reaches some minimum altitude $a$, and then continues with decreasing intensity for time on the order of $\tau_0$:

$$D_d(z, \tau) = \begin{cases} 
0, & \tau < \tau_1 \\
-P_d(z) \exp[-(\tau - \tau_1)/\tau_0], & \tau > \tau_1,
\end{cases}$$

where $\tau_1 = (a - b)/W$. The $P$'s are denoted mass flux profiles.

Figure 1a shows the forms of $P_u(z)$ and $P_d(z)$ chosen for this paper. They differ from the profiles used by R (Fig. 1b) for the following reasons: The updraft profile chosen by R (Fig. 1b, left) implies that only air from below the LFC participates in the updraft, and that it all detrains at the same level, $z = 1$. In reality, updraft mass fluxes seem to increase with height to levels well above the LFC. (See, e.g., Zipser et al., 1981; Ogura and Chen, 1977.) This implies that air of reduced instability from above the boundary layer participates in the updraft. Such air cannot rise to the top of the cloud under its own buoyancy—it must detrain at a lower altitude. The updraft profile used in this paper (Fig. 1a, left) reflects these considerations.

The downdraft profile used here differs from R's in that the downdraft is detrained in a much thinner layer near the ground (see Fig. 1a, b, right). The elevation $a$ at which the updraft begins to form precipitation is set to $z = 0.6$, i.e., the top of the downdraft.

Following Ooyama (1971) we computed momentum and moist static energy forcings assuming that these variables are conserved in vertical currents. Updrafts and downdrafts are treated separately. Thus,

$$F = -\frac{\partial}{\partial z} \{M_u[V(z_u) - V(z)] + M_d[V(z_d) - V(z)]\},$$

$$Q = -\frac{\partial}{\partial z} \{M_u[S(z_u) - S(z)] + M_d[S(z_d) - S(z)]\},$$

where $z_u$ and $z_d$ are, respectively, the levels from which updraft and downdraft air originate. Consonant with the mass flux profiles shown in Fig. 1a, $z_u = 0.15$ and $z_d = 0.45$.

Simple environmental profiles are used in this paper. The LFC is assumed to occur at the top of a boundary layer of variable stability, with constant stability above:

$$N^2 = \begin{cases} 
\frac{z}{b}, & z < b \\
1, & z > b.
\end{cases}$$

(23)

Setting the Brunt-Väisälä frequency to unity above the boundary layer defines the time scale. As illustrated in Fig. 1, the length scale is taken to be the height of cloud top. For a physical $N = 10^{-2}$ s$^{-1}$ and a cloud height of 10 km, the velocity scale is 100 m s$^{-1}$. In terms of potential temperature, the buoyancy scale is the cloud top–surface difference, or about 30 K for a deep cloud.

A wind profile of the form

$$V(z) = V_0 + V_0 z + V_0 z^2/2$$

is assumed for $z < 1$, with a constant wind continuous

FIG. 1. (a) Updraft (left) and downdraft (right) mass flux profiles used in this paper. The level of free convection $b$ can take on any value between 0.1 and 0.3, but was set to 0.1 in this paper. (b) Updraft and downdraft mass flux profiles used by Raymond (1983). The downdraft strength is varied by changing $\gamma$. 
with (24) at higher levels. The advection velocity is assumed to be

$$V_a = V(b),$$

(25)

which says that the steering level of convective cells is the LFC. This assumption is roughly consistent with the results of Marroquin and Raymon (1982) which showed that the steering level for most convective cells was between cloud base and the level of minimum equivalent potential temperature or relatively low in the cloud. ($V_a$ should not be confused with the squall line propagation speed, which is explicitly predicted by the model.)

The assumed ambient profile of moist static energy takes a value of unity for $z = 1$ and $z < b$, and reaches a minimum at $z = z_d = 0.45$, the mean level of origin of downdraft air. Thus,

$$S(z) = \begin{cases} 
1, & z < b \\
0.45 + 0.55 \left( \frac{0.45 - z}{0.45 - b} \right), & b < z < 0.45 \\
0.45, & z > 0.45.
\end{cases}$$

(26)

Equation (6) gives the buoyancy forcing above the LFC. Since (6) is in error for the downdraft below the LFC, downdraft cooling is approximated there by forcing it to be continuous with cooling for $z > b$:  

$$H = N^2 M_u + N^2_M M_d,$$

(27)

where

$$N^2 = \begin{cases} 
N^2(z), & z > b \\
N^2(b), & z < b.
\end{cases}$$

(28)

Integrating the updraft and downdraft parts of (27) separately over $z$ yields, respectively, the net condensational heating and the cooling due to evaporation of precipitation. Minus the ratio of these two quantities is the fraction $f$ of precipitation that is evaporated. Integrating over lags and taking the profiles of Fig. 1a, we obtain

$$f = 0.24 \gamma/(0.3 - b^2),$$

(29)

where $\gamma$ is a measure of the strength of evaporation, defined in Fig. 1a.

3. Physical space simulations

We now develop a method for solving (1)–(5) that avoids the pitfalls of the eigenmode analysis of R, and makes results easier to understand. Since (1)–(5) are linear in $v$, $w$, $\pi$, $\eta$, and $\sigma$, the superposition principle applies and a solution by the Green’s function method is possible.

The key idea is to represent $w_u$ by

$$w_u(x, t) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt' \delta(x - x') \delta(t - t') w_u(x', t').$$

(30)

whence (15) becomes

$$M(x, z, t) = \frac{1}{b} \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt' \delta(z + D(z, t - t'))$$

$$\times \delta(x - x' - V_a(t - t')) w_u(x', t').$$

(31)

Further, representing $w$ and all other dependent variables by equations of the form

$$w(x, z, t) = \frac{1}{b} \int_{-\infty}^{\infty} dx'$$

$$\times \int_{-\infty}^{t} dt' L_u(x - x', z, t - t') w_u(x', t'),$$

(32)

substitution in (1)–(5) yields

$$\left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) L_u + \frac{dV}{dx} L_u + \frac{dL_u}{dz} + \bar{F} \delta(x - V_d t),$$

(33)

$$\frac{\partial L_u}{\partial z} - L_u = 0,$$

(34)

$$\left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) L_u + N^2 L_u = \bar{H} \delta(x - V_d t),$$

(35)

$$\frac{\partial L_u}{\partial x} + \frac{dL_u}{dz} = 0,$$

(36)

$$\left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) L_u + \frac{dS}{dz} L_u = \bar{Q} \delta(x - V_d t),$$

(37)

where $L_u$, $L_u$, $L_u$, $L_u$, and $L_u$ are components of the Green’s function for the subscripted variable. The forcing terms $\bar{F}$, $\bar{H}$, and $\bar{Q}$ are obtained by equating $M_u$ and $M_d$ in (21), (22), and (27) to $D_u/b$ and $D_d/b$, respectively.

Provided (33)–(37) can be solved, (14), (15), and (32) form a closed set of equations yielding the vertical velocity field, the convective updraft and downdraft mass fluxes, and the dispatcher function. In addition, equations analogous to (32) for $v$, $\eta$, $\pi$, and $\sigma$ yield the other dependent variables.

The $L$’s were numerically obtained over a $41 \times 11 \times 51$ grid in $x$, $z$, and $t$ in a manner to be described later. All dependent variables were computed on a $21 \times 11$ grid in $x$ and $z$. The vertical and horizontal grid sizes, $\Delta z$ and $\Delta x$, were respectively set to 0.1 and 1, while the time step $\Delta t$ was set to 2. For the abovementioned length and time scales, the physical grid is $\Delta z = 1$ km, $\Delta x = 10$ km, and $\Delta t = 200$ s. The computational domain is 200 km wide by 10 km deep.

Since $w$ is required to determine $w_u$ at the current time, a two-step iteration is used, with the first guess for the current $w_u$ taken as its value at the previous time step.

Only $w$, $\sigma$, and $M_d$ at $z = b$ were calculated for every time step, since only this information is needed to compute the dispatcher function. All variables at all levels were computed at longer intervals as required. This resulted in a substantial saving of computer time.
Computations were initiated at zero time by an imposed dispatcher function of the form.

\[ w_w = w_0 \exp[-(x - x_0)^2/\chi^2] - w_t, \]  

(38)

where \( w_t \) is set so that the integrated value of the initial \( w_w \) is zero over a region twice the size of the \( x \) domain. Regions where the initial \( w_w \) are negative are ignored in the computation.

Equations (33)–(37) were actually solved in Fourier space and then transformed to physical space. Fourier transforming (33)–(37) in \( x \) yields

\[ \left( \frac{\partial}{\partial t} + ikV \right) \hat{L}_w + \frac{dV}{dz} \hat{L}_w + i k \hat{L}_w = \hat{F} e^{-ikVt}, \]  

(39)

\[ \frac{\partial \hat{L}_w^\prime}{\partial z} - \hat{L}_w = 0, \]  

(40)

\[ \left( \frac{\partial}{\partial t} + i k V \right) \hat{L}_n + N^2 \hat{L}_w = \hat{F} e^{-ikVt}, \]  

(41)

\[ i k \hat{L}_w^\prime + \frac{\partial \hat{L}_w^\prime}{\partial z} = 0, \]  

(42)

\[ \left( \frac{\partial}{\partial t} + i k V \right) \hat{L}_n + \frac{dS}{dz} \hat{L}_w = \hat{Q} e^{-ikVt}. \]  

(43)

Elimination of \( \hat{L}_w^\prime \) between (39) and (40) yields a vorticity equation

\[ \left( \frac{\partial}{\partial t} + ikV \right) \frac{\partial \hat{L}_w^\prime}{\partial z} + \frac{d^2V}{dz^2} \hat{L}_w + ik \hat{L}_w = \frac{\partial \hat{F}}{\partial z} e^{-ikVt}. \]  

(44)

Integration of the vorticity \( \partial \hat{L}_w^\prime/\partial z \) to obtain the horizontal wind \( \hat{L}_n \) requires a boundary condition on \( \hat{L}_w^\prime \). We use the hydrostatic radiation boundary condition of Klemp and Durran (1983) and Bougeault (1983). In present terminology, these authors show that

\[ |k| \hat{L}_w = N \hat{L}_w \]  

(45)

must be satisfied at the upper boundary.

The region \( z > 1 \) is shear and convection free by hypothesis, so (39) and (45) combine to form

\[ \left( \frac{\partial}{\partial t} + ikV \right) \hat{L}_w + i \text{sgn}(k)N \hat{L}_w = 0 \]  

(46)

there. Since there is no density discontinuity at \( z = 1 \), \( \hat{L}_n \) and \( \hat{L}_w \) are continuous, and (46) is thus a prognostic equation for \( \hat{L}_w \) at the top of the domain. This forces a unique solution for \( \hat{L}_w \). Since \( \hat{L}_w = 0 \) at \( z = 0 \), (42) then yields a unique result for \( L_w \).

Integration of (41), (43), (44), and (46) is accomplished by an explicit half-step, full-step technique valid to second order; \( L_w \) and \( L_n \) are diagnosed after both the half-step and full-step.

Successful inverse transformation of the \( \hat{L}_w \)'s requires a certain density and range of \( k \) values. In addition, some smoothing is required to prevent aliasing by very short wavelength waves. This smoothing was accomplished by replacing the delta functions in (33), (35), and (37) with

\[ \frac{1}{(2\pi)^{1/2} \Delta x} \exp[-\nu(x/\Delta x)^2] \]  

(47)

which results in an additional factor of \( \exp(-\Delta x^2 k^2/2) \) on the right sides of (39), (41), and (43). The range of wavenumbers over which the \( L \)'s were computed is given by

\[ k = j \pi/(2l), \quad j = -40, \ldots, 40 \]  

(48)

where \( l = 20 \Delta x \) is the size of the computational domain. This choice makes the smallest wavelength equal to \( 2 \Delta x \) and the repetition length \( 4l \).

Since in numerical approximation a Fourier integral always reduces to a Fourier series, repetition in \( x \) and associated periodic boundary conditions are inevitable. However, experience shows that such a large repetition interval is essentially equivalent to open boundaries for the integration times used here. For the present grid and a length scale of 10 km, the repetition length is 800 km.

An integration time step of

\[ \delta t = \pi/(10k) \]  

(49)

was found to be adequate. This time step is always forced to be less than or equal to \( \Delta t \), the grid interval over which the \( L \)'s are evaluated.

4. Squall line simulations

In this section, examples of both tropical and midlatitude squall lines are simulated. For the midlatitude example I take the Oklahoma case study of Ogura and Liou (1980). This storm moved from west to east across Oklahoma as a north–south line. The wind component normal to the line exhibited shear of about 20 m s\(^{-1}\) over the storm depth. For a scaling velocity of 100 m s\(^{-1}\), this corresponds to a dimensionless shear of 0.2. The wind profile used to simulate this storm is one of constant shear (see Fig. 2c).

As the tropical example the composite of Venezuelan squall lines of Betts et al. (1976) is used. A reasonable approximation to their inflow wind profile is given by Fig. 2d, assuming again a 100 m s\(^{-1}\) scaling velocity.

Both simulations used the profiles of Brunt–Väisälä frequency and moist static energy illustrated in Figs. 2a and b respectively. The LFC was set to \( b = 0.1 \) in both cases, and the updraft rise rate and precipitation time constant were taken to be \( W = 0.05 \) and \( \tau_0 = 20 \). The time constants of both updraft and downdraft were thus about 35 min in dimensional terms, or typical of ordinary convection. The precipitation evaporation fraction was taken to be \( f = 0.5 \), resulting in \( \gamma = 0.6 \). This is in rough accord with Newton's (1966) estimate of the evaporation fraction for an Oklahoma squall line.
level at the top of each plot. Thus, the maximum vertical velocity increases from 0.00105 at time = 20 to 0.00251 at 40, and so on. To the eye this growth would appear as an exponential increase in the areal density of cumulonimbus clouds over a region 50–100 km across. The $e$-folding time is comparable to the assumed updraft and downdraft time constants.

At about $t = 50$ the density of convection becomes sufficient to exhaust the supply of energy in the center of the disturbance. At this point the heretofore stationary disturbance splits into two pieces which propagate away in opposite directions. The speeds of propagation are roughly $\pm 0.1$, or $\pm 10$ m s$^{-1}$ in physical terms, relative to $V_a$ (see Fig. 2c).

If the propagating disturbances are interpreted as squall lines, the foregoing results are reminiscent of those of Moncrieff and Miller (1976) in which a simulated squall line resulted from the decay of an initial region of convection. The obvious difference is that the present model produces two oppositely directed lines, an event that does not appear to occur in real situations. The probable reasons for this are discussed below. The rightward mode moves downslope as is normally seen with squall lines.

Scrutiny of Fig. 2c shows that the rightward mode exhibits a critical level near $z = 0.6$; i.e., the propagation speed of the disturbance coincides with the environmental wind at that level. According to R, this is supposed to be impossible because the critical level interferes with wave propagation from the high level outflow to the surface. However, the present updraft mass flux profile has almost half of its outflow below the critical level, so R’s argument does not apply.

The middle latitude computation was redone with the initiation point offset to $x_0 = 4$. This causes the rightward moving disturbance to be in the center of the computational domain at $t = 100$, while the leftward component disappears out the left side. Stream-

The simulations shared a common initialization, with $w_0 = 0.003$, $x_0 = 10$, and $x_1 = 3$ in (38). This corresponds to a Gaussian upward pulse at the LFC, peaking at 0.3 m s$^{-1}$, with a width of roughly 60 km.

Figure 3 shows the evolution of the dispatcher function over a dimensionless time of 100. For a scaling time of 100 s, this corresponds to about 3 hours. Shortly after initiation, vertical velocities in the disturbance decrease to about $10^{-3}$, or only about 10 cm s$^{-1}$ in dimensional terms. Subsequently an advecting mode begins to grow. Since $V_a = V(b) = 0$ for the wind profile used here, the advecting mode appears stationary.

The growth in amplitude of the advecting mode is clearer in Fig. 4, which shows the vertical wind field at dimensionless time intervals of 20, or roughly 35 min. Note that contours are drawn at percentages of the maximum value, which is denoted the reference level.

![Fig. 3](image_url)
lines were then computed relative to the motion of the right mover. The results are shown in Fig. 5. Also shown are the 30% contours of updraft and downdraft mass flux.

Agreement with the diagnosed streamlines of Ogura and Liou (1980) is reasonably good. In particular, the level of zero relative wind is well predicted, indicating an accurate propagation speed. In both the simulation and the observations the downdraft is fed primarily by middle level air entering the storm from the front (i.e., from the right), with only a slight contribution from inflow from the rear.

Ogura and Liou find a strong downdraft at low levels and an updraft at high levels about 100 km behind the main convective updraft. This is the much-discussed mesoscale anvil circulation. (See reviews by Houze and Betts, 1981; Houze and Hobbs, 1982.)

**FIG. 4.** Vertical wind field in x-z plane at various times for the middle latitude squall line. (a) Time = 20. Note that a downdraft (dry) occurs on the left, or upshear side of the updraft. (b) Time = 40. The amplitude of the updraft has increased by a factor of 2.5 over the previous time, as reflected by an increase in the reference level. (c) Time = 60. The split in the updraft begins. (d) Time = 80. The two propagating modes have separated, leaving an extensive downdraft between them. (e) Time = 100. The propagating modes have moved farther apart and the downdraft has weakened.

**FIG. 5.** Streamlines for the rightward moving disturbance at t = 100. The streamfunction is calculated relative to the disturbance, which is assumed to move at a dimensionless speed of 0.1. The dashed and dotted lines, respectively, show the 30% contours of updraft and downdraft mass flux. The downdraft is clearly fed from the front, or right side of the squall line.
is not reproduced by the present model. The simulated downdraft mass flux occurs much closer to the convective updraft, lagging it by only 20–30 km in dimensional terms. This is consistent with the assumed downdraft time lags, which were purposefully kept in the range of commonly accepted convective time scales, i.e., less than 1 hour. The mesoscale anvil circulation and its associated precipitation are possibly the result of microphysical processes with a time scale of several hours which are not included in the present model.

The results of the tropical simulation were in many ways very similar to the middle latitude computations. Figure 6 shows the evolution of the dispatcher function for the tropical case. Note that the rightward moving disturbance is actually stronger than the left mover. However, the leftward mode now moves with the low level shear. This mode apparently corresponds to real squall lines in the tropics.

Figure 7 shows the streamlines relative to the leftward mode at $t = 100$, assumed to move at the observed dimensionless speed of $-0.095$. In dimensional terms this mode moves about 4 m s$^{-1}$ faster to the left than the jet maximum. This is in good agreement with the observations of Betts et al. (1976). The vertical lines in Fig. 7 indicate the positions of the vertical wind profiles shown in Fig. 8. Reasonable agreement is found between these profiles and the observed mean inflow and outflow profiles. In particular, “westerly” flow is enhanced at high levels by the passage of the squall line, while “easterly” flow is enhanced at low levels.

The tropical and midlatitude simulations differ primarily in the effect of the storm-relative mean wind on the streamline pattern. In the midlatitude case some of the incoming streamlines overturn and exit the front of the storm, whereas in the tropical case all streamlines exit the back of the storm. However, the thermodynamic perturbation fields are quite similar in the tropical and midlatitude cases.

Figure 9 shows the moist static energy for the midlatitude squall line with the streamlines of Fig. 5 overlaid. The system exhibits clear nonconservation of
moist static energy along streamlines in agreement with Ogura and Liou’s (1980) observations. This emphasizes the importance of cross-stream mass exchange.

Figure 10 shows the buoyancy perturbation field for the midlatitude case. The 30% contours of updraft and downdraft mass flux are superimposed as in Fig. 5. The strongest buoyancy perturbations are to the left, or behind the updraft. In terms of potential temperature, the negative buoyancy in the downdraft region at low levels is equivalent to a 1.9°C deficit, whereas the strongest warming just over the downdraft reaches about 1.5°C. Weaker warming on the order of 1°C occurs up to 80 km in front of the squall line as well as in the entire region between the two lines. Weak cooling occurs at high levels. The midlevel warming behind the squall line is in qualitative agreement with Ogura and Liou’s (1980) observations.

5. Diagnostic tests

We now test various aspects of the convective parameterization to determine which are crucial to the maintenance and propagation of the simulated squall lines and which are secondary. Since the tropical and midlatitude simulations exhibited only superficial differences, sensitivity tests are made solely on the midlatitude case.

Observation suggests that updrafts and downdrafts become organized on the mesoscale in squall lines, which implies that they should be explicitly resolvable by the present model. The importance of the unresolved part of the downdraft is tested by turning off the displacement contribution to the dispatcher function, i.e., by setting \( M_p = 0 \) in (14). The restriction associated with exhausting moist static energy is also lifted, so that the results will not be obscured.

The results of such a simulation with conditions otherwise identical to the midlatitude case are shown in Fig. 11. Two propagating modes occur with speeds of ±0.09. These are comparable to, but slightly less than the speeds in the original midlatitude simulation shown in Fig. 3. As expected, no advecting mode occurs. These results suggest that propagating modes are essentially classical wave-CISK modes, with only slight modifications resulting from the combination of displacement forcing and boundary layer exhaustion.

Comparison with Fig. 3 shows another difference; for the same initial conditions, the simulation with displacement forcing is about seven times stronger at \( t = 100 \). Propagating modes tend to amplify rather slowly. This suggests the importance of the initial, rapidly growing advecting mode in producing a strong squall line from weak initial conditions in a reasonable time interval. For purpose of comparison with other simulations in this section, this simulation is denoted the control.

We now examine the relative importance of thermal forcing by the updraft and downdraft. Figs. 12a and b show the dispatcher function for simulations with updraft and downdraft mass fluxes omitted in turn. Forcing by convective momentum transfer is also turned off. All other variables are as in the control.

Downdraft cooling by itself produces modes that move at speeds −0.14 and +0.17. However, these modes decay with time, and are therefore not self-sustaining. Evaporation of a larger fraction of the precipitation might yield positive growth rates, but in any case the modal structure (not shown) is quite odd—both the updraft and downdraft exhibit extreme tilt in the direction of propagation. This is consistent with a gravity wave generated by a low level source such as downdraft cooling, but very different from observed squall lines.

Updraft heating by itself produces a pair of propagating modes moving at speeds of −0.06 and +0.09. Both modes are amplifying. The downshear modal speed is comparable to the control, while the upshear mode moves somewhat more slowly. With no downdraft cooling, the growth rate is also somewhat less than the control.
The updraft and downdraft regions of each are indicated respectively by dashed and dotted lines.

Note that the right hand, or downshear mode has formed a bridge of high moist static energy from low to high levels. In contrast, the upshear mode has been unable to do this in spite of being as strong or stronger in terms of updraft mass flux.

The reason for this difference is made clear by an examination of Fig. 2c. The storm-relative wind at midlevels is much stronger for the upshear than for the downshear mode. Thus, air of high moist static energy deposited at midlevels by the convection is rapidly swept away by the ambient flow. This ventilation effect is much weaker for the downshear mode.

The convective parameterization used here takes no account of the effects of entrainment on the parameterized clouds. However, real clouds are sensitive to entrainment at midlevels (e.g., Mason and Emig, 1961) and local moistening of the environment could make the difference between growth and non-growth. Modes that create a moist environment for the convection would thus be favored.

It is interesting to note that in the tropical case the mode that survives in the real world is the leftward mode. As Fig. 2d shows, this mode has weaker midlevel ventilation than the rightward mode, supporting the foregoing hypothesis.

In the present model, propagating modes typically move at a dimensionless speed of 0.1, or roughly 10 m s⁻¹ in dimensional terms, relative to the wind at the LFC. Weak ventilation at midlevels requires a propagation speed close to the component of the midlevel wind normal to the squall line orientation. The wind component parallel to the line is presumably of less importance, since this just advects moisture down the line.

If midlevel ventilation is significant, this has implications for squall line orientation as a function of
the strength of the wind shear. Fig. 14 shows three schematic examples of linear wind shear and resulting squall line orientation. In the first case, the wind at midlevels is 10 m s\(^{-1}\) greater than the wind at the LFC. This is nominally what is required to make the midlevel wind zero relative to the moving line. In the second case, the shear is greater, and the squall line must be oriented differently to realize zero cross-line relative wind at midlevels. In fact, there are two orientations which satisfy this condition. Both have wind components along the line at midlevels, but in opposite directions. In the third case, the shear is weak and no orientation yields zero ventilation. However, the illustrated orientation minimizes it.

The main consequence of the ventilation hypothesis is that squall lines must always propagate down the low to mid-level shear. However, if the shear is sufficiently strong, the line orientation will not be precisely normal to the shear, but will be rotated so as to exhibit a shear component along the line.

6. Conclusion

The wave-CISK model (Raymond, 1983) with lag effects is implemented as an initial value problem in physical space. Simulations of midlatitude and tropical squall lines are then made, with the following results:

1) Propagation speeds are well predicted for both types of disturbances.
2) Flow structure in the squall front region is also well predicted. In particular, the downdraft appears to feed primarily from the front of the storm in both cases.
3) The mesoscale anvil circulation seen 50–100 km behind many squall lines is not reproduced. This is probably a consequence of inadequate cloud physics in the model.
4) A spurious wave-CISK mode propagating opposite to the real mode, in both cases, is apparently related to the insensitivity of the wave-CISK convective parameterization to midlevel ventilation.
5) The observed explosive development of squall lines is suggested to be a result of the initial, rapidly growing advecting mode.
6) Dissection of the simulation shows that condensational heating in the updraft is the primary forcing mechanism of the simulated squall lines in their mature phase. Evaporative cooling and forcing by convective momentum transfer play secondary roles.

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REFERENCES


—, 1966: Circulations in large sheared cumulonimbus. Tellus, 18, 699-713.


