# THE EFFECT OF BREAKING GRAVITY WAVES ON THE DISTRIBUTION OF TRACE SPECIES IN THE MIDDLE ATMOSPHERE

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ABSTRACT. Gravity wave temperature amplitudes will increase upward into the middle atmosphere due to decreasing ambient density and will grow rapidly approaching a critical level. Waves will break when their amplitudes are so large that they become convectively unstable. Above the breaking level turbulence will mix tracers in the vertical and the mean flow will accelerate toward the phase speed of the wave as the wave is absorbed. A parameterization of these effects is employed in a two dimensional model of the middle atmosphere. Results from this coupled dynamical-radiative-photochemical model are used to review the effects of breaking gravity waves on the mean circulation and on the meridional distribution of trace species.

## 1. INTRODUCTION

Radiative equilibrium temperature calculations do not correctly reproduce the observed cold summer mesopause (160 K) or the relatively warmer winter mesopause (220 K). By using observed temperatures, Murgatroyd and Singleton (1961) showed that radiative heating rates are $\sim$ 10 K/day in the summer mesosphere and $\sim$ -10 K/day in the winter mesosphere. They derived the meridional circulation required to balance this: ascent at $\sim$ 1 cm/s and adiabatic cooling in the summer mesosphere, descent and warming in the winter mesosphere, and a summer to winter flow of $\sim$ 1-5 m/s. Haurwitz (1961) recognized that this meridional circulation is frictionally induced and determined comparable velocities by balancing the Coriolis torque with a diffusion of the zonal wind. Leovy (1964) successfully modeled the observed temperature and zonal wind distributions by decelerating the zonal flow with a Rayleigh friction having a time scale of $\sim$ 10 days. In recent years more detailed radiative calculations (*e.g.*, Wehrbein and Leovy, 1982) have yielded significantly larger heating and cooling rates, which necessitate the use of much larger Rayleigh friction coefficients in numerical models, corresponding to mesospheric deceleration rates of $\sim$ 50-100 m/s-day$^{-1}$ (Schoeberl and Strobel, 1978; Holton and Wehrbein, 1980; Apruzese *et al.*, 1982; Wehrbein and Leovy, 1982; Garcia and Solomon, 1983). By describing the circulation of the middle atmosphere in terms of "vertical motions required to balance heating

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rates" and "drag required to decelerate the zonal flow" one may be left with the impression that the primary cause of the circulation is differential radiative heating. It is now generally agreed, however, that gravity waves are responsible for the reversed winter-summer mesospheric temperature difference (Holton, 1983), greatly accelerate the meridional circulation, and are a major factor in controlling the shapes of zonal jets.

Gravity waves can affect the mean flow in a permanent sense only if they are absorbed. Since time scales for the propagation of gravity wave energy are much smaller than radiative decay time scales, it is believed that wave breaking is the usual precursor to absorption. The concept of gravity wave breaking (Pitteway and Hines, 1963; Lindzen 1967, 1968; Hodges, 1969) is based on conservation of wave action during ascent through shear and decreasing ambient density: eventually wave amplitudes will increase to the point that the local temperature lapse rate becomes superadiabatic. Convective overturning will mix air parcels in the vertical and thereby reduce wave amplitudes. Since wave amplitudes are self-limited above the breaking level, there will be a flux convergence of momentum and the mean flow will be driven toward the phase speed of the wave, which may be far from zero.

Lindzen (1981) introduced a parameterization of breaking gravity waves which allows calculation of vertical profiles of the vertical eddy diffusivity and torque on the zonal flow, given a zonal wind profile and wave phase speeds. Holton (1982) used this parameterization in a numerical model, assumed phase speeds of -20, 0 and 20 m/s in the zonal direction only and obtained a very realistic circulation. Independently, Matsuno (1982) applied a formulation for gravity wave amplitude attenuation due to background turbulence to investigate the effect of steady waves with five discrete phase speeds and four angles of orientation on the mean circulation. Dunkerton (1982) showed that similar results are obtained for transient, stochastic waves. The Lindzen/Holton formulation was employed by Garcia and Solomon (1985) in their two-dimensional model to assess the effect of gravity waves on the meridional distribution of chemical constituents.

The purpose of this paper is to review recent work dealing with the effect of gravity waves on the distribution of trace species in the middle atmosphere in the framework of a two-dimensional model containing interactive dynamics, chemistry and radiation (Brasseur *et al.*, 1987). First we describe the wave breaking parameterization (section 2) and the two-dimensional model (section 3), then present model results in the context of previous studies (section 4). Section 5 contains concluding remarks.

## 2. THE LINDZEN PARAMETERIZATION

Lindzen (1981) considered a gravity wave propagating below its critical level, $z_c$, where the mean flow, $\overline{u}$, equals the phase speed of the wave, $c$. Following Holton (1982) we consider only oscillations in the altitude-longitude plane. If the perturbation vertical velocity is assumed to have the form

$$w'(x,y,z,t) = \tilde{w}(z) \; e^{z/2H} \; e^{ik(x-ct)}, \tag{1}$$

the vertical structure equation becomes

$$\frac{d^2\tilde{w}}{dz^2} + \lambda^2\tilde{w} = 0, \tag{2}$$

where

$$\lambda^2 = \frac{N^2}{(\overline{u} - c)^2} \tag{3}$$

and the symbols have their usual meteorological meaning (*e.g.*, $N$ is the buoyancy frequency). Assuming that the zonal wind varies slowly relative to wave phase, (2) has the approximate WKBJ solution

$$\tilde{w}(z) = \tilde{w}_o \left( \frac{\overline{u} - c}{\overline{u}_o - c} \right)^{1/2} exp \left[ i \int_{z_o}^{z} \lambda dz' \right] \tag{4}$$

where the subscript zero refers to the excitation level. Once generated, a gravity wave will keep the same phase speed relative to the ground. A wave with non-zero phase speed can arise from transient flow over topography, convective activity or 'geostrophic adjustment'. If the wave energy reaches a level where $\overline{u} \rightarrow c$, phase lines come closer together in the vertical ($\lambda^2 \rightarrow \infty$ from (3)) and oscillations tend to become more horizontal ($\tilde{w} \rightarrow 0$ from (4)). Gravity waves stay below their critical level.

The atmosphere is convectively unstable when

$$\frac{\partial T'}{\partial z} + \Gamma \leq 0, \tag{5}$$

where $\Gamma = g/c_p + d\overline{T}/dz$. Alternatively it can be shown that neutral stability is reached when

$$u' = |c - \overline{u}| \tag{6}$$

(Fritts, 1984). Since $T'$ is related to $w'$ through the thermodynamic energy equation

$$-ik(c - \overline{u})T' + \Gamma w' = 0, \tag{7}$$

the altitude at which the wave breaks, $z_b$, can be derived. If the temperature amplitude is large enough some portion of the wave will overturn. Substituting (1), (3) and (4) into (7), differentiating, assuming WKBJ, and removing oscillatory dependences by taking the absolute value, (5) gives the altitude above which the atmosphere is locally convectively unstable:

$$z_b = 3\ H\ ln \left[ \frac{|\overline{u} - c|}{\tilde{u}} \right], \tag{8}$$

if add $z_0 = 15$ km, need to take larger $\hat{u}$ to get similar $z_b$. see Gray and Pyle

where

$$\tilde{u} = \left( \frac{\tilde{w}_o N}{k|\overline{u}_o - c|^{1/2}} \right)^{2/3}. \tag{9}$$

Waves with larger initial amplitudes, longer zonal scales, and those which approach critical levels will tend to break at lower altitudes than other waves. If the formation qualities are parameterized by a specified value of $\tilde{u}$, the breaking height may be calculated for a given phase speed. The wave will break only if $z_b$ is less than or equal to an altitude corresponding to a value of $\overline{u}$ used in (8).

For $z_b \leq z \leq z_c$ wave amplitudes are self-limited by vertical mixing. In this region (7) should include dissipation:

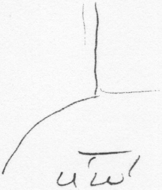
$$-ik(c-\overline{u})T' + \Gamma w' = K_{zz}\frac{\partial^2 T'}{\partial z^2}, \tag{10}$$

from which it can be shown that the vertical eddy diffusivity due to breaking gravity waves is

$$K_{zz} = A\frac{(\overline{u}-c)^4}{N^2}\left[1 - 3H\frac{\partial\overline{u}/\partial z}{\overline{u}-c}\right] \tag{11}$$

where $A = \gamma k/2HN$ and $\gamma$ is the portion of a latitude circle occupied by waves. Since the eddy continuity equation is

$$w' = -\frac{k}{\lambda}u', \tag{12}$$



from (6),

$$\rho\ \overline{u'w'} = -\frac{1}{2}\ \gamma\ \rho\ \frac{k}{\lambda}\ (c-\overline{u})^2 \tag{13}$$

for $z \geq z_b$, so that, using (3),

$$F_g = -\frac{1}{\rho}\frac{\partial}{\partial z}\rho\ \overline{u'w'} = \frac{-k\ \gamma\ (\overline{u}-c)^2}{2N}\left[\overset{\substack{\rho\ \text{decr}\\ \downarrow}}{\frac{(\overline{u}-c)}{H}} - 3\overset{\substack{\text{shear}\\ \downarrow}}{\frac{\partial\overline{u}}{\partial z}}\right] \tag{14}$$

is the body force per unit mass on the zonal flow (Fritts, 1984). By comparing (11) and (14) it is seen that

$$F_g = \frac{N^2}{(c-\overline{u})}K_{zz}. \tag{15}$$

A is a second parameter which, when specified, allows determination of the profiles of $K_{zz}$ and $F_g$. Note that $K_{zz}$ and $F_g$ get smaller approaching a critical level. If a spectrum of gravity waves ascends in westerly shear, westerly waves will not be found above their critical levels, leaving easterly waves to break at higher levels with much higher amplitudes. This will cause easterly acceleration and close off the westerly jet. From (11) it can be seen that $K_{zz}$ should decrease above the jet cores. The expected lower mesospheric maximum in $K_{zz}$ is consistent with the upward decrease near the mesopause derived from atomic oxygen profiles (Brasseur and Offermann, 1986).

## 3. TWO-DIMENSIONAL MODEL

Our model extends from pole to pole and from the surface to 85 km altitude. Chemical species and entropy are advected by a residual (transformed Eulerian) mean meridional circulation formulated in log-pressure coordinates. The residual mean meridional streamfunction is forced by spatial gradients in wave driving and in radiative heating or cooling (see Garcia and Solomon, 1983). Zonal winds are derived from the thermal wind law. In the results reported here, insolation absorption by ozone is calculated with the parameterization of Schoeberl and Strobel (1978), while infrared transfer is approximated by Newtonian relaxation.

The model can be run with a variety of dynamical forcings. Most simply, momentum drag is parameterized by a Rayleigh friction and eddy diffusivities vary with altitude only. A more complex representation of the effects of breaking gravity waves is based on the Holton/Lindzen parameterization. Five phase speeds are chosen and parameters are slightly tuned (see Table I) to yield residual circulations comparable to those inferred from LIMS observations (Hitchman and Leovy, 1986). Profiles of $K_{zz}$ and $F_g$ are smoothed in the vertical to approximate various effects such as nonlinear interaction among waves and radiative damping. A parameterization of Rossby wave driving is also introduced to provide a self-consistent determination of their torque on the zonal flow and meridional tracer dispersion (Hitchman and Brasseur, 1987). The influence of Rossby waves is primarily in the lower stratosphere, but there are feedbacks through the filtering effects of zonal winds on gravity wave transmission.

Approximately 40 species are included. These belong to the oxygen, carbon, hydrogen, nitrogen and chlorine families. The reaction rate constants are taken from the JPL compilation (De More *et al.*, 1985). The diurnal average of the photodissociation coefficients is approximated by a 4 point integral between sunrise and sunset (see Cunnold *et al.*, 1975).

The family grouping technique used to solve the tracer continuity equations avoids the numerical problems associated with the stiffness of the system. Centered space differences are used, while the time integration is done with an implicit 'alternating direction' method (Carnahan *et al.*, 1969).

## 4. MODEL RESULTS

Differential radiative heating during the approach to solstice will cause a weak upward drift in the summer hemisphere, sinking in the winter hemisphere, with Coriolis torques on the summer to winter drift giving rise to summer easterlies and winter westerlies. We expect that westerly gravity waves will be filtered out in the lower winter hemisphere, leaving easterly gravity waves to impart a westward torque on the flow at upper levels. Figure 1 shows the body force per unit mass on the zonal flow due to gravity waves near the austral winter solstice after one year of model integration. Easterly accelerations as high as 80 m/s-day are seen above the westerly jet core (not shown). Near the stratopause the drag is of the order of 20 m/s-day, in good agreement with values derived from the LIMS experiment (Smith and Lyjak, 1985; Hitchman and Leovy, 1986).

These accelerations may be balanced by advection of high angular momentum from the equator to southern latitudes and of low angular momentum from the north pole to northern midlatitudes. Figures 2 and 3 show the meridional circulation induced by gravity wave absorption. The strong meridional winds transport long-lived

Table I. Parameters used to determine the breaking altitudes, vertical eddy diffusivity and momentum drag (eqns. 8, 11, and 14).

| Zonal phase speed (c) [m/s] | Amplitude coefficient (A) [$10^{-9}m^{-2}s^{-1}$] | Breaking level coefficient ($\tilde{u}$) [m/s] |
|---|---|---|
| -40 | 0.25 | 3 |
| -20 | 0.5 | 3 |
| 0 | 1 | 3 |
| +20 | 0.5 | 3 |
| +40 | 0.25 | 3 |

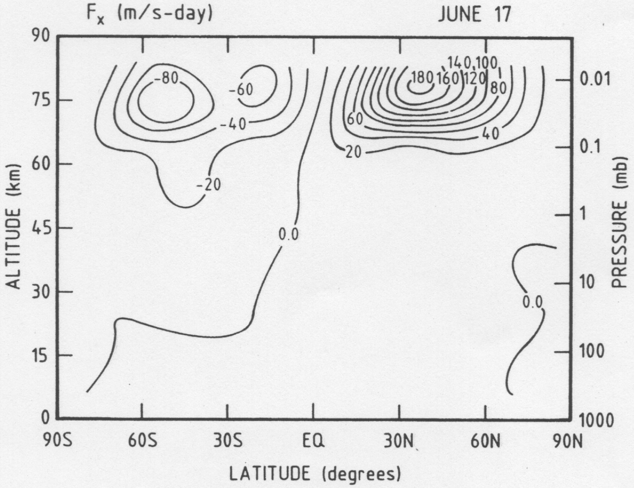


Figure 1. Latitude-altitude section of the body force per unit mass (m-$s^{-1}$-$day^{-1}$) on the zonal flow due to gravity waves near the boreal summer solstice after one year of model integration.

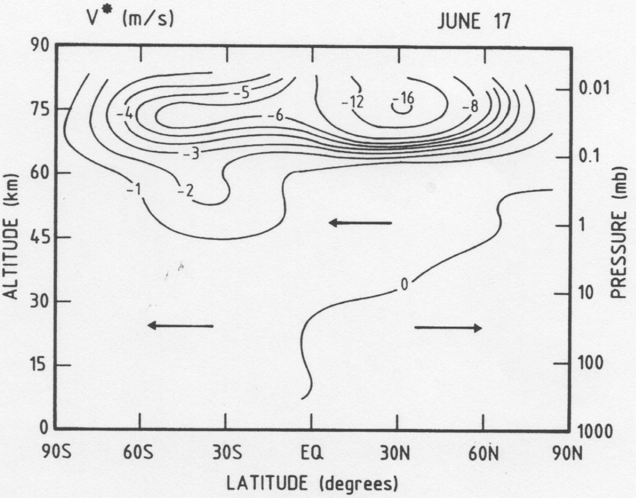


Figure 2. As in Fig. 1, except of the meridional component of the residual circulation (m/s).

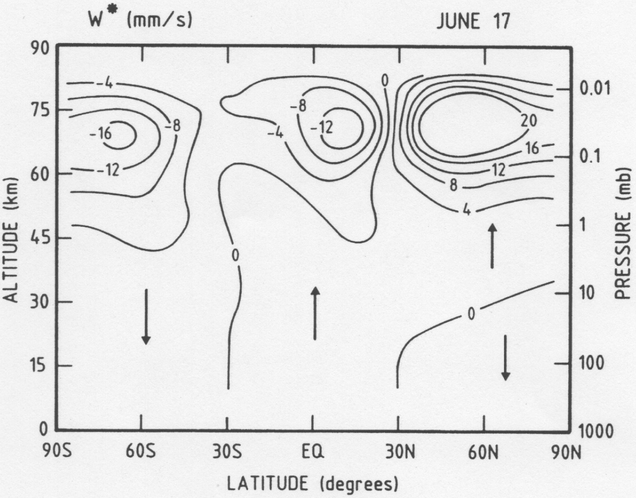


Figure 3. As in Fig. 2, except of the vertical component (mm/s).

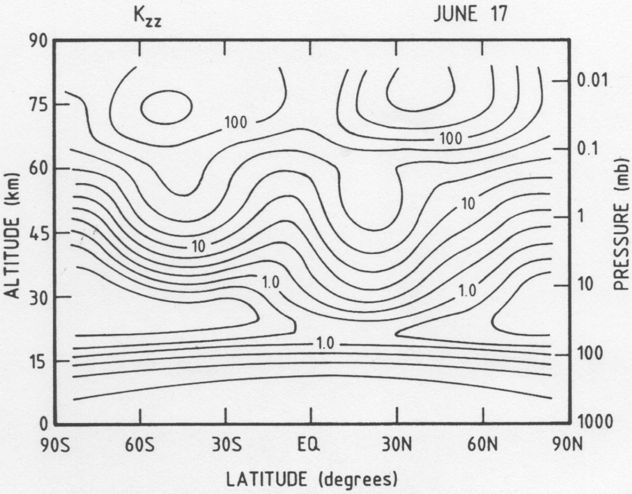


Figure 4. As in Fig. 1, except of vertical eddy diffusivity ($m^2/s$). The values are derived from the Lindzen parameterization (see text).

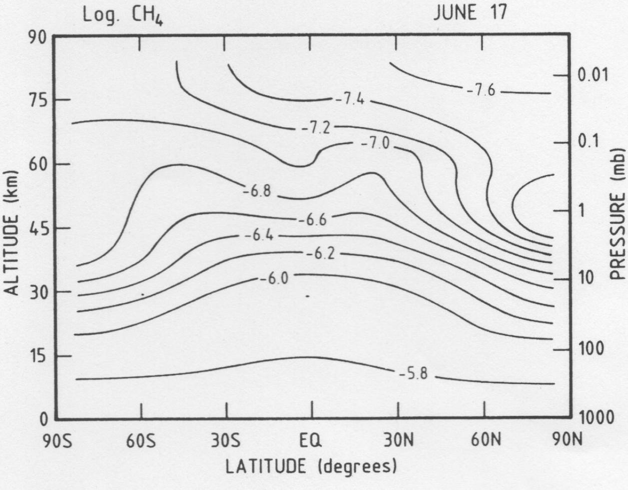


Figure 5. As in Fig. 1, except of the logarithm of methane mixing ratio (-6.0 corresponds to 1 ppmv).

tracers over long distances. It can now be seen that gravity waves force descent over the winter pole, keeping it warmer than it would otherwise be, the region radiating its excess heat to space. Because gravity wave driving is strongest in the summer and winter jets, there is a meridional dipole structure in vertical velocity around each jet. At this time gravity wave driving is strongest in the easterly jet and there is downward motion in the equatorial mesosphere, limiting the upward extent of the 'stratospheric fountain' over the equator. At the solstice the sun angle is changing very slowly. It can be shown by considering the transformed Eulerian mean equations that, in the absence of wave driving, there will be no net heating at the solstice, consequently no meridional circulation. Although the gross structure of the circulation can be obtained by using a global mean vertical profile of Rayleigh friction, important latitudinal and vertical variations cannot be accounted for.

The distribution of $K_{zz}$ due to gravity waves is shown in Fig. 4. As expected, the values are largest near the jets. The strongest vertical mixing due to gravity wave breaking should occur in the midlatitude mesosphere and upper stratosphere. Both vertical advection and the meridional variation in $K_{zz}$ are important in determining the meridional variation of long-lived species such as methane (Fig. 5), nitrous oxide, water vapor, and the chlorofluorocarbons. For each of these species we see a 'double peak' near the stratopause in our model results which is due strictly to gravity wave driving. Such a double peak has been observed in water vapor (Gordley *et al.*, 1985) and in methane and nitrous oxide (Jones and Pyle, 1984). This has been interpreted in terms of advection associated with the semiannual oscillation (Gray and Pyle, 1986; Solomon *et al.*, 1986; Gille *et al.*, 1987). We are currently working on a parameterization of Kelvin waves with the purpose of discerning the relative roles of advection and diffusion in creating this pattern.

Molecules which form in the thermosphere and are stable in the polar night are readily advected downward over the winter pole by the gravity wave driven circulation. Note the large values of odd nitrogen ($NO_y$; Fig. 6) and carbon monoxide (CO; Fig. 7) over the south pole in the upper stratosphere. Nitric oxides are produced in the thermosphere by ionospheric processes and predissociated in the sunlit mesosphere. CO is produced in the lower thermosphere by photodissociation of $CO_2$ and destroyed by reaction with the OH radical in the mesosphere, which is prevalent in the sunlit portion. Near the stratopause the CO mixing ratio is 160 ppbv over the winter pole, compared to 30 ppbv at the equator, while $NO_y$ is 27 ppbv at the winter pole and 8 ppbv in other regions.

As equinox approaches, the zonal jets weaken, gravity wave driving weakens, hence so do the meridional circulation and vertical eddy mixing. Figure 8 shows the distribution of $K_{zz}$ near the vernal equinox, when the latitudinal variation is also significantly less than near solstice. Thomas *et al.* (1984) reported a strong seasonal variation in ozone near 80 km in midlatitudes characterized by large values at the equinoxes and small values at the solstices. Near the equinoxes the reduced upward transport of water vapor would lead to decreased ozone destruction by hydrogen radicals. Such seasonal changes in water vapor in the mesosphere have been observed by Bevilacqua *et al.* (1985). This ozone variation may therefore be attributable to the seasonal variation in gravity wave breaking (Garcia and Solomon, 1985).

Atmospheric tides will produce a diurnally-varying background flow which should strongly influence the time and altitude of gravity wave breaking. Bjarnason *et al.* (1987) have shown that $K_{zz}$ can vary by an order of magnitude during a day, and that the vertical diffusion of species such as atomic oxygen and hydroxyl radicals are significantly modulated by this effect. Such dynamical effects are believed to be important in the diurnal behavior of the airglow emission (Meinel bands).

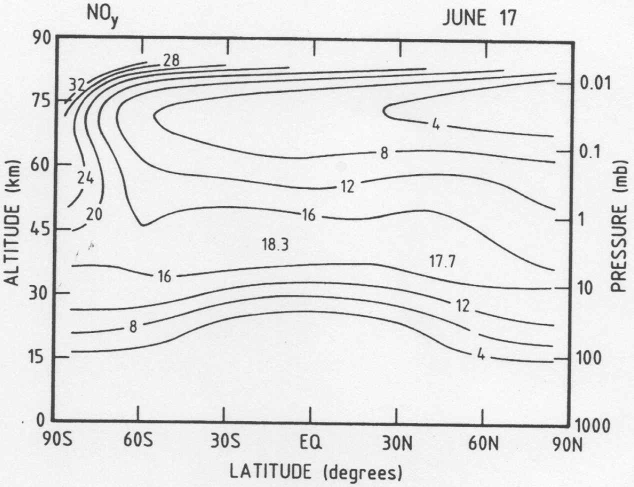


Figure 6. As in Fig. 1, except of total odd nitrogen ($NO_y = N + NO + NO_2 + NO_3 + 2 \times N_2O_5 + HNO_3 + HO_2NO_2 + ClONO_2$) in ppbv.

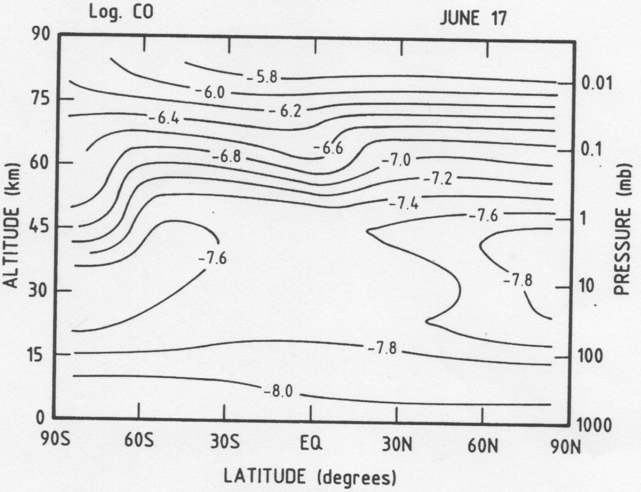


Figure 7. As in Fig. 5, except of carbon monoxide.

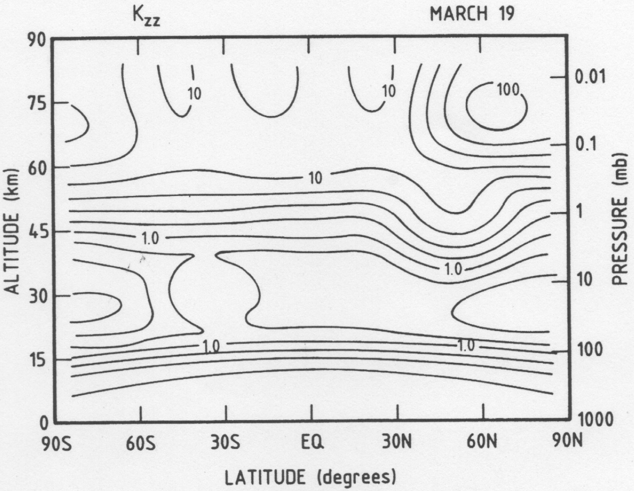


Figure 8. As in Fig. 4, except near the vernal equinox.

## 5. CONCLUDING REMARKS

The distribution of trace species in the middle atmosphere appears to be very sensitive to the distribution of breaking gravity waves, which in turn depends on the distribution of zonal winds and sources of gravity wave action. Since sources are rather variable in space and time, strong variations in species such as water vapor and nitric oxides should be observed in the mesosphere. In the current model source characteristics do not vary with latitude. It is possible that future observations of trace species could help fill the large gap in our understanding of the temporal and spatial distribution of gravity wave sources.

The Lindzen parameterization provides a workable means of estimating the effects of breaking gravity waves which vary self-consistently with the distribution of zonal wind. The parameterization requires specification of phase speeds, source altitudes, initial amplitudes, zonal scales, and occurrence frequencies. The enormous influence of gravity waves on a wide variety of phenomena warrants considerable observational effort toward establishing these values. To explain observed variations in trace species it is important to include the physics of gravity wave breaking in two-dimensional models. In turn, the models themselves may be helpful in establishing gravity wave parameters through tuning to obtain agreement with new observations.

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