AOS 310 Homework #1
Due September 18

You must show all your work. Partial credit is certainly possible even if you get an incorrect solution.

1. Let \( \mathbf{A} = \nabla \phi = 8 \hat{x} + 3y^2 \hat{j} \). If you know that \( \phi(1,1) = 8 \) and \( \phi(0,1) = 4 \), derive a functional expression for \( \phi(x,y) \).

2. Prove the following vector identities letting \( \mathbf{V} = u \hat{i} + v \hat{j} + w \hat{k} \) and \( \nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \)
   
   (a) \( \nabla \cdot (\nabla \times \mathbf{V}) = 0 \)
   
   (b) \( (\mathbf{V} \cdot \nabla) \mathbf{V} = (1/2) \nabla (\mathbf{V} \cdot \mathbf{V}) - \mathbf{V} \times (\nabla \times \mathbf{V}) \)
   
   (c) \( \nabla \cdot (\phi \mathbf{V}) = \phi (\nabla \cdot \mathbf{V}) + \mathbf{V} \cdot \nabla \phi \)
   
   (d) Prove that \( \mathbf{k} \times (\mathbf{k} \times \mathbf{A}) = -\mathbf{A} \) where \( \mathbf{A} = A_1 \hat{i} + A_2 \hat{j} \)
   
   (e) Use the “right hand rule” to verify (d) graphically.

3. The symbol \( A_B \) stands for the projection of vector \( \mathbf{A} \) onto vector \( \mathbf{B} \). In other words, \( A_B \) represents the component of \( \mathbf{A} \) that is parallel to \( \mathbf{B} \). Derive an expression for \( A_B \) in terms of the vectors \( \mathbf{A} \) and \( \mathbf{B} \).

4. Show that a field of pure deformation (i.e. the combination of both components, \( F_1 \) and \( F_2 \)) has no divergence and no vorticity.

5. Consider Fig. 1 which shows isotherms (dashed lines) in fields of pure vorticity, pure convergence (negative divergence), and deformation. The vector \( \nabla T \) has both magnitude and direction.
   
   (a) Do you think the vorticity can change both the direction and magnitude of \( \nabla T \)? Does the orientation of the isotherms affect the answer to the first question? Explain.
   
   (b) Do you think the convergence can change both the direction and magnitude of \( \nabla T \)? Does the orientation of the isotherms affect the answer to the first question? Explain.
   
   (c) Do you think the deformation can change both the direction and magnitude of \( \nabla T \)? Does the orientation of the isotherms affect the answer to the first question? Explain.