AOS 610 GFD I Prof. Hitchman

Key concepts on topics from lectures for the first quiz

1. Flow through or past an obstacle

 pressure gradient force (PGF; Nt) = prime mover

 kinematic viscosity (m^2/s) opposes the prime mover

 dynamic viscosity (kg/m-s) = kinematic viscosity x density

 Reynolds number

 a) ratio of inertia to viscous forces

 b) dynamic similarity of flows with same Re

 c) captures transition to turbulence

 laminar versus turbulent flow

 nondimensionalization

2. Poiseuille flow

 no-slip condition

 steady state

 viscous stress (Nt/m2)

 viscous force (Nt)

 flow curvature required for non-zero viscous force

 momentum diffusion to the wall balances PGF

 Poiseuille Law enables measurement of viscosity

 mass flux = density x speed

 entry length turbulence due to abrupt shear

 turbulent slugs grow into surroundings

 turbulence transports momentum, heat, constituents

 more efficiently than molecules

 if flow is turbulent, to achieve the same increase

 in flow speed a greater increase in PGF is needed

 compared to the laminar flow regime (cf. Fig. 2.11

 in Tritton)

3. Flow past a cylinder

 Re transitions are different than for pipe flow, but still represent

 trend toward greater flow complexity

 Flow creates a drag on an object, which is reflected in diminished flow

 speed in the wake

 At very low Re creeping flow involves symmetric streamlines, with

 the balance between viscosity and PGF. Viscosity is essential for

 drag. Influence of object extends to many L.

 At higher Re attached eddies become unstable and are shed downstream

 At still higher Re turbulent reattachment of the boundary layer reduces

 the size of the wake, thereby decreasing the drag!

 Objects introduce vorticity into the flow

 At small Re drag is proportional to U

 At most values of Re drag is proportional to U\*U (most geophysical situations)

 The drag coefficient is an important nondimensionalization of drag

 which is used in all ocean and atmosphere models to approximate the

 complexity of subgrid scale phenomena

4. Rayleigh Number and convection

 Differential heating gives rise to motions

 Ratio of buoyancy to viscous forces

 Buoyancy force and distance from boundaries promote flow complexity, while

 thermal (kappa) and mechanical (nu) dissipation tend to reduce flow complexity

 Temperature and density perturbations are of opposite sign

 Reduced gravity

 Coefficient of expansion

 At the critical value Ra=1708, buoyancy overcomes viscous dissipation

 and the fluid begins to move. “Benard convection:

 At first roll-like structures transport heat, then at higher Ra

 structures become more polygonal in shape, with increasing time variations

5. Other diffusion concepts

 Prandtl number is the ratio of mechanical to thermal diffusivity (nu / kappa)

 There is a turbulent Pr analogue

 In water the diffusivity of salt is much slower than for heat, giving

 rise to a mixing phenomenon known as salt fingers

 Some buoyancy phenomena are dominated by variations in surface tension

 with temperature or with chemical concentrations (like soap in “color burst”)

6. Link to chaos theory

 For increasing Re or Ra, instabilities lead to turbulence. At low

 Re or Ra dissipation creates a stable attractor. At higher

 values oscillations can occur with a stable limit cycle. At

 still higher values instabilities lead to a Strange Attractor,

 where dissipative processses create preferred regions in

 phase space, but the trajectory in phase space is unpredictable.

 Lorenz (1963) developed his ideas of chaos theory using

 the mathematical equations for Benard convection in a box.

7. Flow kinematics

 Stress/strain

 rate of linear strain

 rate of shear strain

 vorticity

 streamfunction - flow - vorticity (invertibility principle)

 rotational or nondivergent flow

 velocity potential

 divergent or irrotational flow

 Laplacian

 streamline, trajectory, and streakline

 stretching and folding = irreversible mixing

 jet or a wake = vorticity dipole

 vorticity max = streamfunction min

 cyclonic and anticyclonic for NHem and SHem

 vortex tube = material surface for adiabatic frictionless flow

 conservation of vorticity: without viscosity vorticity cannot be introduced;

 vorticity can change following the motion due to vortex stretching or shrinking

 Kelvin’s circulation theorem: In the inviscid limit, circulation is permanent, vorticity

 is “frozen into the flow”

 Types of rotation

 rigid body rotation = no viscous force, constant vorticity at all points = 2 omega

 irrotational vortex = zero vorticity at each point except the center (minus infinity)

 => shear produces filamentation

 acts like angular momentum conserving flow with a mass sink in center

 Rankine vortex is a combination of inner solid body rotation

 and irrotational vortex outside of the jet maximum

8. Boundary layers, separation, and attachment

 Steady Euler’s equation shows that udu/dx ~ -dp/dx; as speed increases pressure decreases

 P + ½ rho u^2 = constant along a streamline, so wind slowing to zero on the side of a building

 exerts a dynamic pressure

 Pressure is highest on the upstream end of an object, lowest on the sides, where the speed

 is fastest, and medium in the wake

 Fluid entering a constriction moves faster, so pressure decreases downstream

 Fluid entering an expansion moves slower, so pressure increases downstream

 Irrotational flow impinging on an object introduces vorticity into the flow via viscosity

 A boundary layer is characterized by the same sign of vorticity. It grows with the square root

 the distance downstream and is inversely proportional to flow speed

 Pressure decreasing downstream favors boundary layer attachment;

 pressure increasing downstream favors boundary layer growth and separation

 airplane wings introduce paired vorticity into flow, related to lift and contrail formation

 Sports balls curve when the spin drags the boundary layer with medium pressure to one side, but

 low pressure remains on the other side, with a PGF perpendicular to the direction of motion

9. Continuum hypothesis

 Below a certain scale successive samples of molecules yield varying

 values for speed, temperature, etc., so there is a physical limit

 below which fluids really can't be considered continuous.

 Collision cross-section

 Mean free path

 At larger space and time scales there will be variation within a volume,

 so we get into the problem of inadequate sampling of real variability,

 which undermines forecasting and diagnostic capabilities

 The atmosphere and ocean exhibit "red noise" spectra in the sense that

 variance tends to get larger at longer space and time scales, a characteristic

 aspect of climate and climate change.

 To study a global change problem, scales from molecular to global must

 be included, with chemical lifetimes varying from less than 1 s to

 centuries, another major challenge.

 There is an intimate link between smallest and largest scales.

A primary goal is to derive the Navier-Stokes equations.

The gradient of surface stresses gives rise to the Surface Forces on a fluid element. How do we relate the deformation to stress? We need a fourth order tensor to do this.

10. Tensors

 Defined by how they rotate

 Their order is defined by the number of free indices

 Zeroth order - scalars, e.g. Temperature

 First order - vectors

 Second order - stress tensor (3 directions on each of 3 faces to a cube)

 velocity gradient tensor (same)

 Kronecker delta

 Third order - alternating or permutation tensor, used to define 3D vorticity

 Fourth order - 81 element matrix that governs rotation of a second order tensor. Each element could be a nonlinear function of who knows what.

 Contraction: set two indices equal to obtain the trace of a matrix

 The velocity gradient tensor has symmetric and antisymmetric parts:

 symmetric part includes divergence (trace; ignored) and shear strain elements (off-diagonal)

 antisymmetric part includes vorticities, which do not deform a fluid element and are ignored in defining the constitutive relationship

11. Newtonian fluid approximation and the Navier-Stokes equations

 Since the velocity gradient tensor is the motion of the boundaries of the fluid element, it makes sense that the forces on the fluid element are intimately linked to the velocity gradient tensor. In deriving the Navier-Stokes equations it is assumed that the 4th order tensor that links the two reduces to a constant, while the divergent part of the symmetric part of the velocity gradient tensor is set to zero (incompressibility applied to deriving the viscous stress only).

 The normal components of the stress tensor give rise to the pressure gradient force

 The tangential components of the stress tensor give rise to the viscous force

 Some fluids, like polymer molecules or emulsions (suspended particles) exhibit a nonlinear viscosity with increasing shear or viscoelastic memory. Corn starch in water does not seem to respond linearly with increased shear, rather it becomes more like a solid when stressed.

 Using empirical molecular diffusivities is a strength and a blessing for the

Navier-Stokes equations

 Net force on a fluid element is given by gravitational acceleration, pressure

gradient force, and viscous force

 Assumptions: charge neutral fluid, continuum, incompressibility for viscous force, Newtonian linear coefficient relating shear to stress

 Viscous force applies to each velocity component with the 3D Laplacian

 Equations are useful for predicting flow evolution at any point in domain, but need to have predictive equation for temperature (hence pressure) to capture the

process of differential heating giving rise to motion through buoyancy.

12. Mass Conservation

 Eulerian derivation with fixed volume V and surface S

 Using substantive derivative can go back and forth between Eulerian and Lagrangian forms

 Lagrangian derivation for fixed mass in a deforming fluid element gives same relation

 Density=constant (incompressibility) is a good approximation when the Mach number squared is small. Ma=U/c, where c is the speed of sound. Ma^2 = KE/IE. So incompressibility is a good approximation when wind or wave speeds are much less than the speed of sound. Even so, we need the elemental prime mover of buoyancy production in many problems, so temperature perturbations are retained in the buoyancy term for these situations.

13. Boussinesq approximation

 density = constant except in buoyancy term

 linear perturbation method

 In the vertical momentum equation the imbalance between gravitational and the vertical pressure gradient force is re-written as "reduced gravity" dependent on rho’ or T’in the derivation the quadratic perturbation term and p’ terms are ignored, while the basic state hydrostatic balance is subtracted out

 This approximation eliminates sound waves

14. Trace Species Conservation Equation

 Gauss' Theorem

 Eulerian fixed volume or Lagrangian approach gives

 local time rate of change = advection + divergence term + sources/sinks

 Even in the absence of sources/sinks, a scalar will not be conserved

 following the motion if there are significant vertical displacements.

 Volume mixing ratio IS conserved following the motion in the absence

 of sources and sinks.

 Mass mixing ratio = volume mixing ratio x Molecular weight / Mol. Wt. air

 Photochemical time scale = amount / loss processes

 Dynamical time scale = length scale / speed scale

 The "Chemistry limit": local time rate of change = sources and sinks

 (applies when dynamical time scale is very long compared to

 chemical time scale; example: OH, which has a lifetime of less than one second)

 The "Dynamics limit": local time rate of change = advection

 (applies when dynamical time scale is short compared to

 the chemical time scale: example: N2O, which lives 180 years)

 The most interesting problems are those where chemistry and dynamics

 are both important.

 AOS models often require simulation of constituents with lifetimes