AOS 610 GFD I Prof. Hitchman

Key concepts from lectures since the first quiz

1. Rotation

Reading suggestions: Gill 4.5; Kundu 4.11; Holton 2.1, 2.2, 2.3, 1.5

Extra reading if you really want to dig into it: Simon "Mechanics" 3.5;

Stommel and Moore (1989) "An Introduction to the Coriolis Force"

By posing Newton's Law in a rotating coordinate system on a sphere we obtain

a) Centripetal acceleration, folded into "gravity" g

b) four Coriolis terms

c) five curvature terms, due to rotating unit vectors

Relation between acceleration in the fixed and rotating frame

Gravitational potential and centrifugal potential

Conservation of angular momentum, Moment arm

Synoptic scale motions

Ferrel 1859: hydrostatic and geostrophic large scale flow

Acceleration and friction are needed to achieve geostrophic or hydrostatic balance.

Rossby number = acceleration/Coriolis = U/fL, is small for synoptic scale flow

Read Gill 7.6 on geostrophic balance

Know geostrophic balance in terms of pressure gradient or height gradient

Brandes 1820: wind and pressure distributions related

Birt 1847: wind and pressure distributions propagate as a wave pattern

Gradients of ocean surface or geopotential height are closely related to currents or winds.

2. Vertical Coordinates and Thermal Wind

Pressure is the weight of molecules above you.

Geopotential is the work done against gravity.

Geopotential height is geopotential per unit mass divided by gravity at sea level and is close to geometric height. Thickness between pressure surfaces is proportional to the mean temperature in the layer.

Scale height = RT/g.

Z = - H ln P/Po is "log pressure coordinates"

P = Po exp(-Z/h)

Geostrophic wind components depend on pressure gradients on a constant height surface or height gradients on a constant pressure surface. They both slope the same way so we ignore the - sign in dp=-rho g dz in going back and forth.

Constant height maps have isobars on them. Constant pressure charts have lines of constant altitude on them.

Thermal wind law: Cold toward pole = westerly shear, cold toward west = poleward shear. Jet streams overlie strong temperature constrast. Jet maxima are overlain by a reversed temperature gradient.

3. Equation of State

State variables = pressure, temperature, composition

Atmosphere

78% N2, 21% O2, 1% Ar, 0-4% H2O

gas constant for dry air

Dalton's law of partial pressures

ideal gas law

virtual temperature

Water vapor is less than 40/1000 parts of the atmosphere but makes a huge difference for where rising and sinking occurs. More vapor means lighter air.

Ocean

Density depends on pressure, temperature, and salinity

Salinity includes 55% Cl, 30% Na, 8% SO4, 4% Ma, 1% K, 1% Ca, and many others

Salinity variations are less than 40/1000 but they make a huge difference for where rising and sinking occurs.

Range for 98% of oceans: salinity 28.5-37 pptm, temperature -2C to 30C, density 1017-1055 kg/m3

Potential density is density that water would have if it were brought adiabatically to 10 m depth. Isopycnals plotted in temperature-salinity space can be compared with local soundings of density to evaluate vertical stability and likelihood of convection, but you need to make sure that you compare isopycnal patterns at the same depth as the observations.

The Atlantic gets salty in the subtropics by evaporation, cools in the north part of the Gulf Stream, and gets even saltier from brine rejection upon sea ice formation. North Atlantic bottom water forms in narrow negatively-buoyant chimneys. Near 11,000 years ago the steadily rising temperatures reversed and oscillated, possibly due to the "North Atlantic Oscillator Mechanism" involving glacial meltwater, Laurentide ice sheet position relative to the St. Lawrence seaway, and shutting off the Gulf stream. This requires the presence of the

Laurentide Ice Sheet to work. Variability in the North Atlantic has been linked to a variety of global phenomena at various time scales. A smaller amplitude version of this occurs in the Holocene, but again with periodicities of ~1200 years. An emerging theory is that there is a fundamental charge-discharge time scale in the ocean involving alternate bottom water formation in the North Atlantic and circumpolar Antarctic, a “bipolar see-saw”.

4. First Law of Thermodynamics

Kinetic energy K

Internal energy I

Potential energy P

Chemical energy C

Stored energy equation (I+K)

Kinetic energy equation (K) comes from wind dot Navier-Stokes' equation

Total energy equation (K+I+P+C)

Thermal energy equation (I) or temperature equation

To obtain the T equation, subtract the kinetic energy equation from the stored energy equation: following the motion temperature can change if there is compression/expansion, chemical heating, heat flux divergence, or viscous dissipation (the last three are called "diabatic processes")

Viscous dissipation always increases I and decreases K

Work by expansion decreases I

Work by surface stresses and gravity only affect K, not I

Heat flux convergence and latent heat only affect I

In steady state plane Couette flow energy must be put into keeping the top plate moving, which goes into the surface work term, and ultimately into increasing the temperature, even in the absence of acceleration!

In the atmosphere I~73%, P~25%, K~2%

Available Potential Energy ~ temperature variance

The general circulation is due to:

Differential heating -> temperature differences -> density differences -> kinetic energy -> viscous dissipation; internal energy increases

5. Second Law of Thermodynamics

The change of entropy, S, for a closed system is greater than or equal to zero.

The time rate of change of S equals net heating divided by temperature.

If there are no diabatic processes then entropy is conserved.

We introduce a new variable to describe entropy, potential temperature

theta, where dS = Cp d ln (theta)

Both the 1st and 2nd laws have net heating as one term, so we can equate

changes in theta to changes in temperature and pressure.

Memorize theta = f(T, P), and theta = f(T, z)

Since dT/dt and dTheta/dt have units of K/s, it is important to be

clear about which it is, since they are not numerically equal.

Isentropic motion

Since net heating is usually less than 2 K/day (imbalance of radiation

to space and latent heat release), parcels away from the planet's surface will

not drift very far from a theta surface for several days.

In the lower world, isentropes connect the subtropical sea surface with

the extratropical upper troposphere, along which water vapor is transported

and precipitated out.

In the middle world isentropes connect the upper troposphere and lower

stratosphere, allowing for the possibility of strat/trop exchange if

there is mixing by waves.

In the upper world isentropes are completely in the stratosphere and

net heating rates are a few tenths of a degree per day, so the isentropic

approximation for parcel motions is good for perhaps a week.

Conserving dry static energy yields the adiabatic lapse rate of 10 K/km,

while including latent heat release in the form of moist static energy

yields the moist adiabatic lapse rate, which averages about 6.5 K/km.

The buoyancy frequency, or Brunt-Vaisalla frequency, N, is the restoring

coefficient for vertical parcel displacement. N~.02 per s in the

troposphere, so the buoyancy period is about 7 minutes. It is shorter

in the stratosphere and longer in the ocean. If the displacement is

sloping instead of vertical, the restoring force of gravity projected

onto that direction is weakened, so gravity wave periods are always

longer than the buoyancy period.

Convective systems can be analyzed for their thermodynamic efficiency

as a Carnot engine. If the tropopause is higher and colder, then the

storm can be more efficient in converting latent heat release into KE.

6. Vorticity, Circulation, and Potential vorticity

For solid body rotation circulation divided by the area of a circle is 2 omega.

If density is constant then the circulation can’t change. If it does then there a “solenoidal term” which can change the circulation (e.g. sea breeze).

Relative vorticity is the circulation divided by area. Planetary vorticity is the Coriolis parameter. Absolute vorticity = relative plus planetary vorticity.

A vorticity equation can be formed by taking d/dx of the meridional momentum equation minus d/dy of the zonal momentum equation.

For incompressible flow at synoptic scales, absolute vorticity will not change unless there is divergence/convergence, or stretching/shrinking. This motivates defining potential vorticity, which most generally is static stability x absolute vorticity (Ertel’s PV).

PV is useful for representing cyclones and anticyclones. PV is quasi-conserved

on theta surfaces. A positive PV anomaly on the tropopause is

a developing cyclone, with low static stability and consequent mixing.

A negative PV anomaly near the tropopause is an anticyclone, with high

static stability and suppressed vertical mixing. Negative PV anomalies

may last longer than positive PV anomalies.

For a single layer fluid of depth h we have shallow water PV: (zeta + f) / h.

Conservation of zeta + f gives the Rossby wave propagation mechanism, where beta is df/dy. Conservation of zeta / h in a rotating tank can give Rossby waves. Conservation of f / h helps to describe the shallowing of ocean gyres toward the equator. A reversed PV gradient (dP/dy < 0)tells you when Rossby waves break and irreversibly mix.

7. Turbulence is characterized by a) random, b) nonlinear, c) vorticity,

d) dissipation, and e) efficient mixing. When energy is injected at a

large scale to create a vortex, smaller and smaller vortices are created,

eventually leading to viscous dissipation at the molecular level. This

is captured by L. F. Richardson's 1922 poem:

Big whorls have little whorls

that feed on their velocity.

Little whorls have lesser whorls

and so on to viscosity.

This seems to have been modeled after Jonathan Swift's 1733 poem:

So naturalist observe, a flea

hath smaller fleas that on him prey;

And these have smaller still to bite 'em

and so on ad infinitum

Thus every poet, in his kind,

is bit by him that comes behind.

Richardson observed seagulls over the oceanic boundary layer and concluded

that viscosity depends on length scale to the 4/3 power! This was

confirmed theoretically by Kolmogorov (1941) and separately by Obukhov.

At scale l\* deformation work by shear strain associated with turbulence

equals diffusion by molecular viscosity. Below this scale turbulence

is not apparent. Above this scale turbulence dominates. l\* depends

on viscosity to the 3/4 power and on the KE dissipation rate per unit mass

to the 1/4 power.

Taking precipitation minus evaporation or solar heating minus infrared cooling

averaged over a region, there is typically about 25 W/m2 available

for generating KE. Dividing by 8000 m for an air column of density 1 kg/m3

yields a required dissipation rate of .003 m2/s3, hence l\* ~ 1 mm.

More intense turbulence will make l\* smaller, such as in the planetary

boundary layer. In the free atmosphere away from turbulence sources

l\* can be larger.

Power spectra. Energy flow to small scale turbulence goes fast in 3D

turbulence, where vortex stretching in 3D readily leads to vortex-vortex

interaction. This is the inertial subrange, with a k^(-5/3) slope. If one

plots k E(k) or omega E(omega), then equal areas represent equal power.

In 2-D turbulence, energy transfer among scales is inhibited, and the spectral

slope is steeper, with a k^(-3) slope.

8. Momentum fluxes and Wave Drag

The Reynolds stress arises in treating subgrid-scale effects of waves

and turbulence, such as estimating water vapor flux in the boundary layer. Eddy momentum fluxes for zonal mean flow. One can form a diffusivity using mixing length theory, but this applies to diffusion by isotropic turbulence, and is degraded information relative to fluxes, which includes information about the orientation of wave phase axes and non-isotropy. A famous example is Victor Starr's realization that Rossby waves can flux momentum into the subtropical westerly jet, implying a negative viscosity or eddy diffusion coefficient!

Extras: A Collection of Fluid Dynamics Principles

\*Bernoulli's equation, Pitot tube, Venturi tube

For steady, adiabatic, incompressible flow, conservation of stored energy

yields the Bernoulli relation, which conserves dry static energy along

a streamline.

As upstream flow U slows to a stagnation point in front of an object,

the local pressure (stagnation pressure) is the sum of the regular

thermodynamic pressure plus the dynamic pressure 0.5 rho U^2.

This is consistent with Euler's equation, which relates deceleration

to downstream pressure increase and acceleration to downstream

pressure decrease. It also shows that defining a pressure coefficient

by dividing pressure by the dynamic pressure is useful.

Pressure distribution around the surface of an obstacle: highest upstream,

lowest on the sides where the flow is fastest, and medium in the wake.

\*Kelvin's Circulation theorem

If Euler's equation holds (part of this is to assume constant density)

then the circulation around a closed contour cannot change.

If initially the flow is irrotational, it will remain so.

An object introduces vorticity into the flow through the no-slip condition

and the formation of a boundary layer. A given boundary layer is

therefore characterized by a uniform sign of vorticity. If one sees

a change in the sign of the vorticity, there are two different boundary

layers present.

The thickness of the boundary layer grows as the square root of the

distance downstream. When the flow profile at each point along the object

is normalized by the local boundary layer thickness, the flow profiles

all collapse to a single shape, the Blasius profile.

\*Separation and attachment

If pressure decreases downstream the flow will accelerate and the boundary

layer thickness decreases. In this favorable pressure gradient a

boundary layer will stay stuck to the object. A flow constriction can

cause this.

If pressure increases downstream the flow will decelerate and the boundary

layer thickness will increase. If this adverse pressure gradient is

strong enough the flow will reach a stagnation point, beyond which

the original boundary layer detaches from the object and a new boundary

layer is formed characterized by vorticity of the opposite sign.

This can happen in a flow expansion.

A jet is fundamentally unstable and will break into turbulence, which

entrains air around it, so a jet will spread downstream. If the

spreading reaches a wall it will become a boundary layer attached to the

wall.

A jet aimed across the top of an object will create low pressure there.

If it is strong enough it can suspend the object against gravity.

This is known as the Coanda effect.

The drag on an object includes viscous drag by the surface and form drag

due to the distribution of pressure around the object. The existence

of form drag depends on the formation of viscous boundary layers, so

viscosity is essential in the form drag component also.

\*Airfoils

Drag parallel to the motion and lift perpendicular to the motion are

coupled to an actual object by the drag and lift coefficients.

From Kelvin's circulation theorem, if there is no initial circulation

around a wing at rest on the runway, then a starting vortex must

be left over the runway to exactly compensate for the circulation

around the wing that causes lift.

Due to the sharpness of the trailing edge of a wing, air flows farther

over the top of the wing in the same time as going under the wing,

so there is a non-zero circulation. The Kutta-Zhukovskii theorem

gives the lift per unit span as - rho \* U \* circulation.

Due to the pressure on the bottom of a wing exceeding that on the top,

air flows upward around the wingtips, creating trailing vortices that

are consistent with downwash in the wake. The downward acceleration

of air in the downwash must equal the force of gravity on the plane

to keep it aloft.

As the angle of attack of the wing increases, the drag coefficient

can increase to the point of exceeding the lift coefficient, causing

a dangerous stall.

\*Sports Balls

A spinning ball can swing the wake to one side, creating medium pressure

there, with low pressure on the other side, causing a pressure force

perpendicular to its direction of motion. This is a curve ball.

A ball with ridges thrown without spin can irregularly excite boundary

layers which lead to motions perpendicular to the direction of flight.

This is a "knuckleball".

This effect is called the Robins effect or the Magnus effect.

Due to boundary layer reattachment it is possible for some sports balls

to curve the opposite way to that described above.

Dimensional Analysis

Dynamical similarity is useful for developing prototypes, studying GFD

problems in tanks, diagnosing flows, predicting behavior, and

eliminating small terms.

Some nondimensional numbers to know are

pressure coefficient

drag coefficient

Reynolds number

Rayleigh number

Prandtl number

Froude number

Richardson number

Mach number

Rossby number

Fr^(-2)=Ri

Fr measures the importance of inertia relative to gravity

Ri is useful for indicating whether convection will occur.

For motions in the interior of a fluid, reduced gravity is used

and we have the internal Froude number.

Dimensional analysis is useful for checking if an equation is correct,

or at least is self-consistent for units on both sides. It can be

used to create nondimensional or dimensional numbers, such as the

Kolmogorov microscale.

Equations can be nondimensionalized and cast in the form of terms of

varying order in specific nondimensional numbers. This readily shows

which terms can be neglected for a given magnitude of nondimensional

number.

Buckingham's Pi theorem:

If you have n variables describing a situation, 3 of the variables are

used to take care of the fundamental dimensions of time, length, and mass.

So n-3 products can be formed that are nondimensional.

When one equates the exponents of each "dimension" of time, length, and mass,

one finds the power of each variable required to form a nondimensional

number.