E. L. DEACON. I think it is a very useful departure to relate τ_0 and other features to the general meteorological conditions in terms of the geostrophic wind.

G. I. TAYLOR. I was very interested to see these results as they emphasize the point which I worked on years ago and I am glad to see the study continuing. I found that a treatment of the distribution of velocity, using a uniform K, gave a ratio of nearly 0·6 and a corresponding angular change at the ground. It also led to a drag coefficient of the same order of about 0·002.

B. BOLIN. These results will be of great interest to those studying the dynamics of large-scale flow patterns at high elevation. It has always seemed to me that the kind of friction acting on these flow patterns is determined by transport set up by the vertical gradients occurring in the free atmosphere rather than by the detailed conditions at the surface. It may be that in these cases we can partly determine the frictional drag from the internal characteristics of the atmosphere rather than from the detailed structure at the earth's surface.

G. K. BATCHELOR. How long does it take for the stress at the ground, and the accompanying small-scale features of turbulence, to become established after a certain geostrophic wind is set up? I see Lettau's point for a completely steady situation but I am not sure that the flow near the ground will respond sufficiently rapidly to a change of geostrophic wind for the steady state to be set up very often.

H. LETTAU. It is dependent on the altitude and there may be complications due to inertial oscillations. Except in conditions of extremely rapid change, say near fronts, I think the adjustment will be very rapid close to the ground but will take longer in the upper part of the boundary layer.

G. K. BATCHELOR. But I thought you were claiming that the geostrophic wind, i.e. the upper air conditions, were the cause and ground stress the effect—I should have thought that the changes would begin from the top and be propagated downward, as in the case of a change of speed in the stream outside the boundary layer on a flat plate.

H. LETTAU. I hope Batchelor will work out some theory for this; I am merely going on the experimental data which are available.

H. R. BYERS. Of course, it is clear that the geostrophic wind was used because it is known and can be predicted.

P. A. SHEPPARD. It is, of course, important to bear in mind the type of problem we are dealing with. For large-scale dynamical meteorology there is a good deal to be said for Lettau's approach, but for an understanding of the momentum transfer problem it would be dangerous. After all, the velocity V_g does not necessarily exist, as does u_* i.e. $(-\overline{u'w'})^{1/2}$. It is merely a pressure-field parameter, and the relationship between it and the turbulence field and momentum transfer must be very involved and indirect, particularly in a changing pressure field.

H. LETTAU. I agree that V_g is a fictitious quantity, but then we could substitute for this the horizontal pressure gradient, which is a physical reality.

A. M. OBUKHOV. It is very interesting to have these macroscopic relationships for studies of the general circulation.

R. W. STEWART. Very close to the ground we normally assume that the flow is driven by the stress gradient and not by the pressure gradient. It seems to me that Lettau's point is that the stress is imposed rather independently of the nature of the boundary.

H. LETTAU. The sequence of the physical effects is wind first and stress next.

R. W. STEWART. But the velocity close to the ground is controlled essentially by stress gradient. The stress does not respond quickly to the pressure gradient.

E. L. DEACON. This is certainly true of the first 30 m. above ground, but over rather deeper layers the pressure gradient becomes important.

G. I. TAYLOR. The real point seems to be that z_0 does not make much difference to V/V_G.

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Lettau

WIND PROFILE, SURFACE STRESS AND GEOSTROPHIC DRAG COEFFICIENTS IN THE ATMOSPHERIC SURFACE LAYER

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SUMMARY

An important problem in meteorology is to establish relationships between the turbulence characteristics of the lower atmosphere and the large-scale synoptic parameters. A significant turbulence characteristic is the surface stress or ground drag which determines—together with the roughness length of the surface—the wind profile, the eddy diffusivity, and the local energy dissipation in the atmospheric surface layer. A significant large-scale synoptic parameter is the geostrophic speed as derived from the horizontal pressure gradient near the earth's surface. The geostrophic drag coefficient is defined which determines also the ratio of wind speed in the surface layer to the geostrophic speed. Its dependence on surface roughness (Surface Rossby Number) is investigated, and compared with similar drag relationships in fluid mechanics (duct flow). The effect of thermal stratification (Richardson number) on surface stress and wind profile structure is also discussed.

1.

The Reynolds stress τ (dynes/cm²) is a meteorological variable which holds a key position for several important problems of the atmospheric boundary layer ($0 \leqslant z \leqslant H$), where z = height (m., or cm.) and H is a level (of the order of 1,000 m.) which can be taken as the top of the atmospheric boundary layer. Let ρ = air density (g/cm³), V = mean horizontal wind speed (m/sec., or cm/sec.), and τ_0 = ground drag = boundary value of τ. In cases of neutral thermal stratification the micro-meteorological profiles of mean wind speed V and mean shear $V' \equiv \partial V/\partial z$ (sec⁻¹) in the lowest stratum of the atmospheric boundary layer ($0 \leqslant z \leqslant h \ll H$, where h = height of the atmospheric surface layer, which is of the order of 10 m.), are known to be satisfactorily expressed by the relations

$$(1) \qquad V = k^{-1}(\tau_0/\rho)^{1/2} \ln (z+z_0)/z_0, \qquad V' = (\tau_0/\rho)^{1/2}/[k(z+z_0)],$$

where k = universal Kármán constant and z_0 = roughness length (cm.) which is introduced in Equation (1) so that the lower boundary condition $V = 0$ for $z = 0$ is satisfied. The above equations imply that an eddy diffusivity K (cm²/sec.) exists which is defined by

$$K = \tau_0/\rho V' = k(z+z_0).(\tau_0/\rho)^{1/2}, \quad \text{for } 0 \leqslant z \leqslant h. \tag{2}$$

In Equations (1) and (2) the quantity $(\tau_0/\rho)^{1/2}$ can be considered as the fundamental measure of resistance, or diffusion power, at any given level of the atmospheric surface layer.

For studies of turbulence structure the local kinematic value of energy dissipation $\epsilon = V'\tau/\rho$ (cm²/sec³) is significant. In the atmospheric surface layer, Equation (1) gives

$$\epsilon = V'\tau_0/\rho = (\tau_0/\rho)^{3/2}/[k(z+z_0)], \quad \text{for } 0 \leqslant z \leqslant h. \tag{3}$$

From the vertical distribution of ϵ in the entire atmospheric boundary layer ($0 \leqslant z \leqslant H$), the total dissipation E (ergs cm^{-2}sec^{-1}, or watts/m²) is obtained,

$$E = \int_0^H \rho\epsilon\, dz = \int_0^H \boldsymbol{\tau}.\boldsymbol{V}'\, dz. \tag{4}$$

The quantity E has great importance for both synoptic meteorology and the theory of the general circulation since it represents the major portion of the continuous energy decay in large-scale air currents.

Equations (1) to (4) are listed here in order to illustrate the significant role played by the Reynolds stress in a variety of atmospheric flow problems ranging from turbulence structure to micrometeorology to macrometeorology.

It seems desirable that techniques be explored which permit us to estimate the value of the Reynolds stress in the lower atmosphere above a given location and for a given time, on the basis of certain general geophysical information, that is to say, with the aid of certain external parameters of atmospheric boundary layer flow. The roughness length z_0 in Equation (1) can be considered one such parameter. Namely, it is generally assumed that the topography of a micrometeorological field site, or the physical and vegetational structure of the earth/air interface, determines uniquely the local z_0 value. This statement is supported by the fact that tabulations of empirical representative z_0-values for various types of natural surface can be found in the general textbooks of micrometeorology; see, for example, Sutton (1953, p. 233). There are at least two other significant parameters of a given atmospheric flow type. These are the Coriolis parameter $f = 2\omega \sin\phi$(sec^{-1}) and the representative horizontal pressure gradient $\partial p/\partial n$ (dynes/cm³), where n = horizontal coordinate normal to the isobars, ω = angular speed of the earth's rotation, and ϕ = geographic latitude. A substitute for the horizontal of the diameter of the elements of wall roughness. The symbols used in the various relationships are explained on both graphs.

It can be seen that the above-defined drag coefficient varies from approximately 0·02 to 0·05 when the scale factors which represent a combination of the significant external parameters range over approximately 5 powers of ten.

3.

Turning to atmospheric boundary layer flow the first problem is to establish the significant scale factor which is a non-dimensional combination of the external parameters. On the basis of theoretical reasoning, Rossby and Montgomery (1935, p. 17) were first to note the importance of the ratio $V_g/(z_0 f)$ for the problems of frictional influences in wind and ocean currents. During the last decade the ratio $V/(Lf)$, where L is a characteristic horizontal scale of a given meteorological process, has come to be known as the Rossby number. In order to avoid confusion it is here suggested to call the original form of this ratio, with the roughness parameter z_0 as the characteristic length, the "Surface Rossby Number",

$$Ro_0 = V_{g,0}/(z_0 f). \qquad (6)$$

With the aid of simultaneous observations, or estimates of ground drag, roughness length, and geostrophic speed, it is possible to investigate the empirical relationship between the geostrophic drag coefficient, Equation (5), and the Surface Rossby number, Equation (6). The number of cases for which such a complete set of variables and parameters is reported in the literature is unfortunately not very large.

In order to avoid complications which can arise from the presence of convective-type turbulence, it is advisable to restrict the study to cases of purely, or at least dominantly, mechanical turbulence, that is to say, to measurements or estimates of τ_0 and z_0 for neutral thermal stratification or the adiabatic atmosphere. The geostrophic drag coefficient is then denoted by C_a where the subscript refers to adiabatic conditions.

Actual data used for the computation of C_a for the sites indicated on Fig. 3 are mainly taken from Halstead *et al.* (1957), Sheppard and Omar (1952), and Lettau (1957a, 1957b). The two relatively extreme values for drag on forests were derived from a re-analysis of wind profile data reported by Baumgartner (1956) in combination with estimates of geostrophic wind speed on isobaric maps of the German Weather Service. However, it is felt that the available information is by no means satisfactory and reliable; in particular, the points on Fig. 3 for extreme Ro_0-values are quite uncertain. A curve of best fit was computed with the aid of the method of least squares and is illustrated on Fig. 3. This empirical curve follows a mathematical form as suggested by Prandtl's resistance law depicted on Fig. 1,

$$C_a = 0 \cdot 104/[\log_{10}(C_a\, Ro_0) - 2 \cdot 24]. \tag{7}$$

Since the values of the numerical constants are not very certain, they must be considered tentative and subject to future revision, provided that more and improved observations become available. However, it seems to be relatively safe to state that the geostrophic drag coefficient

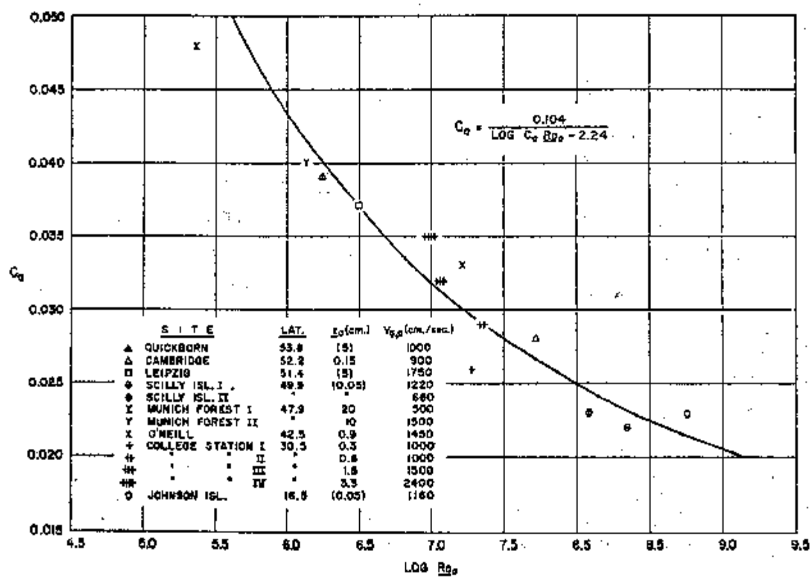

FIG. 3. Geostrophic drag coefficient versus Surface Rossby number.

C_a depends systematically and significantly on the Surface Rossby number and varies approximately from $0 \cdot 02$ to $0 \cdot 05$ when Ro_0 ranges over at least 4 powers of ten. Thus, an interesting similarity between the results plotted on each of the Figs. 1, 2, and 3 exists which appears to be more than pure coincidence. In view of the uncertainties of data on Fig. 3 a detailed discussion may be postponed until more reliable information is available.

The practical significance of Equation (7), or possibly any improved form of this type can be readily demonstrated. For example, the combination of Equation (5) with Equation (1) yields

$$V/V_{g,0} = (C_a/k) \ln(z + z_0)/z_0. \tag{8}$$

prediction is obvious. Reference can be made to Mintz (1958), who estimated a geostrophic surface stress coefficient from the horizontal flux and surface sinks of vorticity. Mintz assumed a greatly simplified relation in which the ground drag is directly proportional to the geostrophic speed at the surface. This may be considered as a crude approximation of the hydrodynamically more satisfactory quadratic relation given by Equation (5). Within a limited range of conditions both estimates do agree with each other. While Mintz compressed all effects of external parameters into a single dimensional quasi-constant, the explicit formulation of the prominent external parameters in Equation (7) will perhaps be especially useful for an application of such relationships in numerical forecasting experiments.

7.

In conclusion it must be remarked that the foregoing presentation is given with the chairman's suggestion in mind that the controversial side of the subject be stressed. An attempt was made to follow this suggestion, in order to stimulate a discussion by the participants of the Symposium. It is realized that there is much to be desired concerning accuracy and reliability of the empirical data—see Fig. 3—on the geostrophic drag coefficient, which is to be considered as the basic concept in this presentation. Both experimental and theoretical aspects of the problem remain to be clarified; primarily, of the role played by surface heating or cooling, and the non-steadiness and non-uniformity of the horizontal pressure gradient, in determining the turbulence structure of the atmospheric boundary layer.

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